Errata for “Time-stamps for Mazurkiewicz traces”

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Abstract

This is an errata for [1].

The following elementary lemma can be proved by induction on the length of traces.

Lemma 1 *Let $t \in \mathbb{M}(\Sigma, \mathscr{D})$, $a \in \Sigma$ and $\emptyset \neq A \subset \Sigma$ be such that $a \notin A$ and $\partial_a(\partial_A(t)) \neq \mathbf{1}$. Then there exists a sequence $c_0, \ldots, c_n$ of elements of Σ such that*

- $c_0 \in A$, $c_n = a$ *and*
- $(c_i, c_{i+1}) \in \mathscr{D}$ *and* $\partial_{c_{i+1}}(\partial_A(t)) \sqsubset \partial_{c_i}(\partial_A(t))$ *for all* $0 \leq i < n$.

Lemma 5 (c) of [1] should be replaced by the following one

Lemma 2 *Let $t \in \mathbb{M}(\Sigma, \mathscr{D})$, A, B non-empty subsets of Σ. If*

$$\mathbf{1} \neq \partial_a(\partial_A(t)) \sqsubset \partial_a(\partial_B(t)) \tag{1}$$

then there exists $c \in \Sigma$ such that

$$\partial_a(\partial_A(t)) \sqsubset \partial_c(\partial_A(t)) = \partial_c(\partial_a(\partial_B(t))) \sqsubset \partial_a(\partial_B(t)) \ . \tag{2}$$

PROOF. Note that since $\partial_a(\partial_A(t)) \sqsubset \partial_a(\partial_B(t)) \sqsubseteq \partial_a(t)$ we have $a \notin A$.

Thus by Lemma 1 there exists a sequence $c_0, \ldots, c_n$ of elements of Σ such that $c_0 \in A$, $c_n = a$, $(c_i, c_{i+1}) \in \mathscr{D}$ and $\partial_{c_{i+1}}(\partial_A(t)) \sqsubset \partial_{c_i}(\partial_A(t))$ for $0 \leq i < n$. Note that since the first element of this sequence is in A while the last not the sequence (c_i) contains at least two elements. Moreover all c_i are pairwise different since the corresponding traces $\partial_{c_i}(\partial_A(t))$ are all different.

Let

$$m = \min\{i \mid \partial_{c_i}(\partial_A(t)) \sqsubseteq \partial_a(\partial_B(t))\} \ .$$

Note that m is well-defined since the set $\{i \mid \partial_{c_i}(\partial_A(t)) \sqsubseteq \partial_a(\partial_B(t))\}$ contains at least one element: $c_n = a$, cf. (1). We shall show that $c = c_m$ satisfies (2).

Applying ∂_{c_m} to both sides of

$$\partial_{c_m}(\partial_A(t)) \sqsubseteq \partial_a(\partial_B(t))$$

we get

$$\partial_{c_m}(\partial_A(t)) = \partial_{c_m}(\partial_{c_m}(\partial_A(t))) \sqsubseteq \partial_{c_m}(\partial_a(\partial_B(t))) \ . \tag{3}$$

We shall prove that in fact we have the equality

$$\partial_{c_m}(\partial_A(t)) = \partial_{c_m}(\partial_a(\partial_B(t))) \ . \tag{4}$$

Suppose that the prefix relation (3) is strict, i.e.

$$\partial_{c_m}(\partial_A(t)) \sqsubset \partial_{c_m}(\partial_a(\partial_B(t))) \ . \tag{5}$$

Then m cannot be equal to 0 since $\partial_a(\partial_B(t)) \sqsubseteq t$ and $c_0 \in A$ imply $\partial_{c_0}(\partial_a(\partial_B(t))) \sqsubseteq \partial_{c_0}(t) = \partial_{c_0}(\partial_A(t))$ and (5) would not hold. Thus c_{m-1} exists and from the definition of the sequence (c_i) we get

$$\partial_{c_m}(\partial_A(t)) \sqsubset \partial_{c_{m-1}}(\partial_A(t)) \ . \tag{6}$$

Since $(c_{m-1}, c_m) \in \mathscr{D}$, $\partial_{c_m}(\partial_a(\partial_B(t)))$ and $\partial_{c_{m-1}}(\partial_A(t))$ are non-empty prime traces comparable by the prefix relation. In fact, since $c_{m-1} \neq c_m$, these traces cannot be equal and we have either

$$\partial_{c_m}(\partial_a(\partial_B(t))) \sqsubset \partial_{c_{m-1}}(\partial_A(t))$$

or

$$\partial_{c_{m-1}}(\partial_A(t)) \sqsubset \partial_{c_m}(\partial_a(\partial_B(t))) \ .$$

In the first case $\partial_{c_m}(\partial_a(\partial_B(t))) \sqsubset \partial_{c_{m-1}}(\partial_A(t)) \sqsubseteq \partial_A(t)$ implies $\partial_{c_m}(\partial_a(\partial_B(t))) = \partial_{c_m}(\partial_{c_m}((\partial_a(\partial_B(t)))) \sqsubseteq \partial_{c_m}(\partial_A(t))$ contradicting (5). In the second case $\partial_{c_{m-1}}(\partial_A(t)) \sqsubset \partial_{c_m}(\partial_a(\partial_B(t))) \sqsubseteq \partial_a(\partial_B(t))$ contradicting the definition of m. This terminates the proof of (4).

Now note that

$$c_m \neq c_n = a \ . \tag{7}$$

Indeed $c_m = a$ and (4) would imply $\partial_a(\partial_A(t)) = \partial_{c_m}(\partial_A(t)) = \partial_{c_m}(\partial_a(\partial_B(t))) = \partial_a(\partial_a(\partial_B(t))) = \partial_a(\partial_B(t))$ contradicting (1).

However, since $\partial_a(\partial_B(t)$ is a non-empty prime trace, (7) implies that $\partial_{c_m}(\partial_a(\partial_B(t)))$ is a proper prefix of $\partial_a(\partial_B(t)$:

$$\partial_{c_m}(\partial_a(\partial_B(t))) \sqsubset \partial_a(\partial_B(t) \ . \tag{8}$$

Similarly, (7) yields $m < n$ implying that

$$\partial_a(\partial_A(t)) = \partial_{c_n}(\partial_A(t)) \sqsubset \partial_{c_m}(\partial_A(t))$$

which together with (8) and (4) proves that $c = c_m$ satisfies the thesis. □

Now it remains to modify accordingly the proof of Proposition 8 of [1]:

Proposition 3 (Proposition 8 of [1]) *Let $t \in \mathbb{M}(\Sigma, \mathscr{D})$, $a \in \Sigma$, A, B two non-empty subsets of Σ such that $\partial_a(\partial_A(t)) \neq \mathbf{1} \neq \partial_a(\partial_B(t))$. Then $\partial_a(\partial_A(t)) = \partial_a(\partial_B(t))$ if and only if $\lambda(\partial_a(\partial_A(t))) = \lambda(\partial_a(\partial_B(t)))$.*

PROOF. The left to right implication follows just from the definition of λ.

Suppose that $\partial_a(\partial_A(t))) \neq \partial_a(\partial_B(t)))$. Since the traces $\partial_a(\partial_A(t)))$ and $\partial_a(\partial_B(t)))$ are comparable by the prefix relation, without loss of generality we can assume that $\partial_a(\partial_A(t))) \sqsubset \partial_a(\partial_B(t)))$. By Lemma 2 there exists $c \in \Sigma$ such that $\partial_a(\partial_A(t))) \sqsubset \partial_c(\partial_A(t))) = \partial_c(\partial_a(\partial_B(t)))) \sqsubset \partial_a(\partial_B(t)))$. Then, by Lemma 6 of [1] and by definition of λ, $\lambda(\partial_a(\partial_A(t)))) \prec \lambda(\partial_c(\partial_A(t)))) = \lambda(\partial_c(\partial_a(\partial_B(t))))) \prec \lambda(\partial_a(\partial_B(t))))$, i.e. $\lambda(\partial_a(\partial_A(t)))) \neq \lambda(\partial_a(\partial_B(t))))$. □

References

[1] W. Zielonka. Time-stapms for Mazurkiewicz traces. *Theoretical Computer Science*, 356(1-2):255–262, 2006.