UPDATE to
Random Reals à la Chaitin
with or without prefix-freeness

July 16, 2008

Theorems 2.4 and 2.7 assume the hypothesis

$$A \textit{ is partial many-one } \Sigma^0_n \textit{ complete}$$

This hypothesis is, in fact, equivalent to

$$\begin{array}{rl} & n = 1 \textit{ and } A \textit{ is nonempty} \\ \textit{or} & n \geq 2 \textit{ and } A \textit{ is many-one } \Sigma^0_n \textit{ complete} \end{array}$$

The proof of this equivalence is an easy consequence of Proposition 2.2 and the following facts from Mohrherr's paper [1] :

1. Let $(\varphi_x)_{x \in \mathbb{N}}$ be a standard enumeration of partial computable functions $\mathbb{N} \to \mathbb{N}$.
   - If $A \subseteq \mathbb{N}$, let $A^{pm} = \{(x, y) \mid \varphi_x(y) \in A\}$.
   - B is a pm-cylinder if it is recursively isomorphic to some A^{pm}.
   - A set is dual-pm if A and $\mathbb{N} \setminus A$ are both pm-cylinders.
2. The set $\emptyset''$ is dual-pm *(cf. Mohrherr's paper, page 832, §3 line 5, where* $\emptyset''$ *is denoted by* K'*).*
   Relativizing, this is also valid with $\emptyset^{(p)}$, for $p \geq 2$.
3. If A is dual-pm and is partial many-one equivalent to B then A and B are recursively isomorphic *(cf. Mohrherr's paper, page 833, Proposition 5).*

Observe that $\emptyset^{(p)}$ is many-one Σ^0_{p+1} complete. Thus, if B is partial many-one Σ^0_{p+1} complete ($p+1 \geq 2$) then it is recursively isomorphic to $\emptyset^{(p)}$ hence many-one Σ^0_{p+1} complete.

References

[1] J-L. Mohrherr. Kleene index sets and functional m-degrees. *J. Symbolic Logic*, 48(3):829–840, 1983.