PhD Research Proposal

Mealy machines, automaton (semi)groups, decision problems and random generation

Position

The position is a 36 month full-time doctoral position at LIAFA (Laboratoire d'Informatique Algorithmique: Fondements et Applications), UMR 7089 CNRS & Université Paris Diderot. It is tied to a project entitled **MealyM** funded by the French ANR (JCJC 12 JS02 012 01). The thesis will be co-advised by I. Klimann and M. Picantin.

Context and motivation

The broad research area of this proposal is the theory of automaton (semi)groups. Algebraic structures like groups or semigroups are objects of choice for mathematicians, while objects like automata or transducers are historically used and studied by computer scientists. This distribution of roles would be a cliché if we did not mention that for several decades these two fields had shown stronger and stronger links. The upcoming book chapter by Bartoldi and Silva gives a rich demonstration [2]. The aim of this research proposal is nothing less than adding a new (double) link between mathematics and computer science via the automaton (semi)groups.

Computer science for mathematics: decision problems on automaton (semi)groups. Traditionally in computer science, automata are used and studied as recognizing words, series and trees. Automata as generating (semi)groups have been used and studied mainly by mathematicians, especially geometric group theorists. This construction has proved its efficiency by giving example and couter-example to some difficult mathematical problems, such as Burnside or Milnor problems. There are still pending a lot of decision questions on these (semi)groups. The aim is to bring new ideas from theoretical computer science to this domain.
Some first results [1, 4, 5, 6, 7] tackle the finiteness problem for automaton (semi)groups, which automata-theoretic tools are applied on. In the project under consideration, we intend to conclude the decidability of the finiteness problem for wider and wider classes of automaton groups, to further enhance the associated algorithms and to analyze other related decision problems, in particular the order problem and the growth problem. We do not exclude to pursue ideas which would lead to results on property problems like deciding whether some Mealy machine generates a nilpotent, a solvable, a free, a periodic, a finitely presented (semi)group, all these features having to be considered in parallel with the second axis below.

Mathematics for computer science: random generation of (automaton) (semi)groups. Efficient algorithms that generate random combinatorial objects have been proposed in the last decades. "Random Generation" is now an established part in the computer science field of "Analysis of Algorithms", in which researchers analyze the average complexity of algorithms and the typical properties of large random data structures. Random generation is mostly used to test the robustness of programs, to help in determining the average complexity of an algorithm and to assist the researcher when testing conjectures.
The choice of the distribution is a major issue in random generation. Naturally, from one distribution to another, the typical properties of objects can differ a lot. When having specific applications in mind, the distribution is chosen to model the data properly. But when testing the robustness of an implementation or looking for counterexamples to a conjecture, a random generator producing very different objects is to be preferred. This is the kind of distributions we aim at in this project.
Random group have received considerable attention since the pioneer work of Gromov. A survey of Ollivier [8] gives a picture of the known results in 2005. Most distributions lead to "typical properties", that is, group properties that holds with very high probability for a randomly chosen group. This is not very satisfactory to test performances of programs or to look automatically for counterexamples.

Objectives

The goal of this research is to develop an original approach to the class of automaton (semi)groups in order to make significant contributions to specified decision problems and random generation. Therefore the objectives are twofold: on the one hand, to obtain theoretical results on automaton (semi)groups using computer science techniques, mainly answering decision problems in an effective way: does such a Mealy machine generate a finite or an infinite (semi)group? what is the type of growth? what is the order of some element of the group? etc. A great success would be an algorithm deciding finiteness for automaton groups or an algorithm computing the order of elements in a bireversible automaton group.
On the other hand, to produce efficient tools to generate random (semi)groups via random Mealy machines. We propose to investigate the properties of random automaton (semi)groups by choosing distributions on their associated automata. The first experiments look promising since it seems that such distributions are quite different from the ones of the literature, yielding a wide diversity. A great success would be the design of a fully parametric random generator for (semi)groups whose systematic usage helps in finding new computer algebra solutions for (semi)groups.
Results obtained in both directions will be included into Sage, directly or through GAP [3].

Eligibility criteria

The candidate must have a background in one of this fields:
- mathematics (including group theory)
- theoretical computer science (including autamata theory or semigroup theory)

Application process

Thank you for sending your application form containing:
- a CV
- a cover letter
- marks and ranking during graduate studies
- letters of recommendation

to `klimann@liafa.univ-paris-diderot.fr` and `picantin@liafa.univ-paris-diderot.fr`

References

[1] A. Akhavi, I. Klimann, S. Lombardy, J. Mairesse, and M. Picantin. "On the finiteness problem for automaton (semi)groups". *International Journal of Algebra and Computation* 22.6 (2012). http://arxiv.org/abs/1105.4725.

[2] L. Bartholdi and P. V. Silva. "Groups defined by automata" (2010). http://arxiv.org/abs/1012.1531.

[3] *GAP – Groups, Algorithms, and Programming, v.4.7.2.* "http://www.gap-system.org".

[4] P. Gillibert. "The finiteness problem for automaton semigroups is undecidable". http://arxiv.org/abs/1304.2295.

[5] I. Klimann. "The finiteness of a group generated by a 2-letter invertible-reversible Mealy automaton is decidable". *Proc. 30th Symposium on Theoretical Aspects of Computer Science* (STACS 2013). http://arxiv.org/abs/1208.6324.

[6] I. Klimann, J. Mairesse, and M. Picantin. "Implementing computations in automaton (semi)groups". *Proc. 17th International Conference on Implementation and Application of Automata* (CIAA 2012). http://arxiv.org/abs/1310.4856.

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[8] Y. Ollivier. *A January 2005 invitation to random groups*. Vol. 10. Ensaios Matemáticos, Sociedade Brasileira de Matemática. http://www.yann-ollivier.org/rech/publs/randomgroups.pdf.