Exercises

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1 Exercise 1

Consider the automaton $\mathcal{A}$ represented in Figure 1.

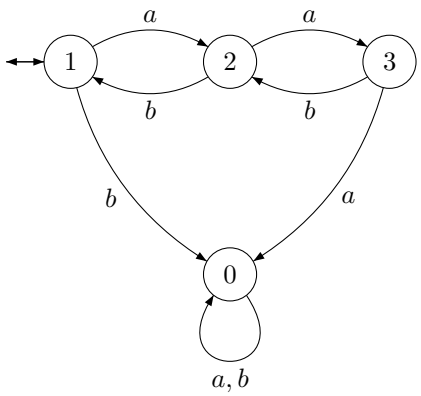

Figure 1: The automaton $\mathcal{A}$.

Give a rational expression for the language L recognized by $\mathcal{A}$.

2 Exercise 2

Compute the transition monoid M of the automaton $\mathcal{A}$ (Hint: you should find 15 elements).
What are the idempotents of M ?
Is M an aperiodic monoid ? Is it commutative ?
Is L star-free ? Is it commutative ?

3 Exercise 3

Can L be defined by a first order formula ? Justify your answer.

Solution

4 Exercise 1

$(a(ab)^*b)^*$

5 Exercise 2

The transition monoid M is

			1	2	3
$*$	1	1	1	2	3
	2	a	2	3	0
	3	b	0	1	2
	4	aa	3	0	0
$*$	5	ab	1	2	0
$*$	6	ba	0	2	3
	7	bb	0	0	1
$*$	8	aaa	0	0	0
	9	aab	2	0	0
	10	abb	0	1	0
	11	baa	0	3	0
	12	bba	0	0	2
$*$	13	$aabb$	1	0	0
$*$	14	$abba$	0	2	0
$*$	15	$bbaa$	0	0	3

The relations are

$$aba = a \qquad bab = b \qquad bbb = 0 \qquad aaa = 0$$
$$baaa = 0 \qquad baab = abba \qquad aabba = aab \qquad abbaa = baa$$

The idempotents are $\{1, ab, ba, aaa, aabb, abba, bbaa\}$.
This monoid is aperiodic but not commutative. Therefore L is star-free, but not commutative.

6 Exercise 3

By McNaughton's theorem, every star-free language can be defined by a first order formula.