A Brief History of Real-Time

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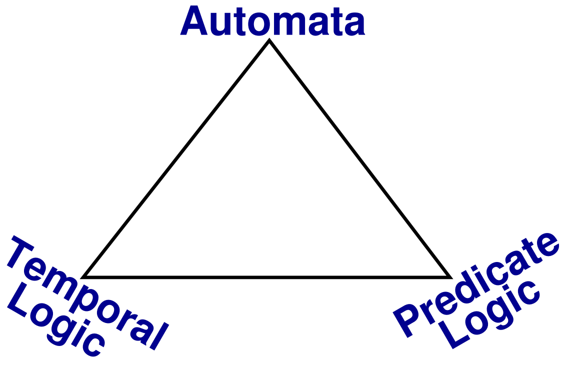
EQINOCS Workshop, Paris, May 2016

The Classical Linear Theory of Verification



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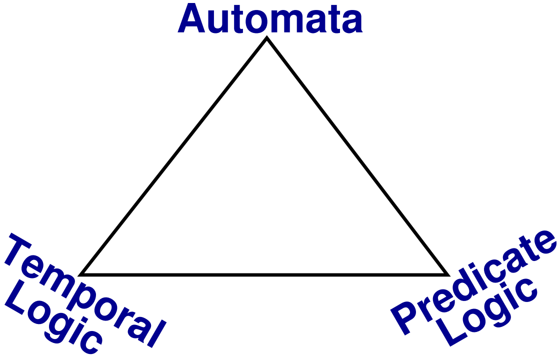

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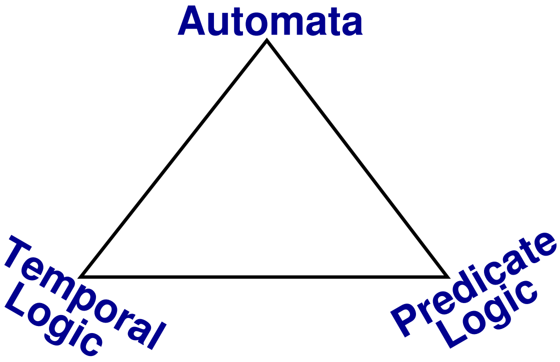

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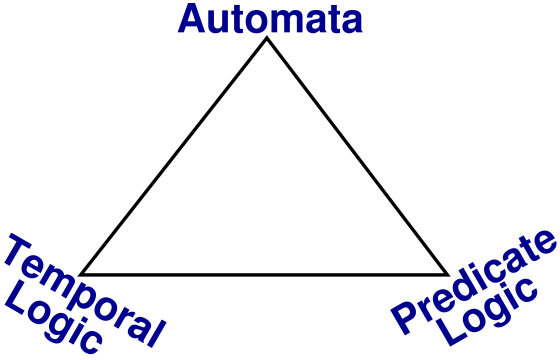

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- Note: focus on linear time (as opposed to branching time).

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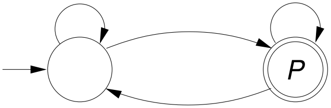

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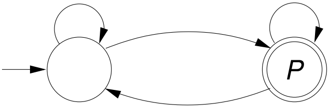

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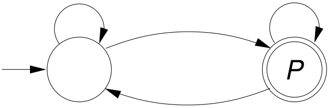

$\Box\Diamond P$ $\qquad\qquad$ $\forall x\, \exists y\, (x < y \wedge P(y))$

Specification and Verification

- Linear Temporal Logic (LTL)

$$\theta ::= P \mid \theta_1 \wedge \theta_2 \mid \theta_1 \vee \theta_2 \mid \neg\theta \mid \bigcirc\theta \mid \Diamond\theta \mid \Box\theta \mid \theta_1\, \mathcal{U}\, \theta_2$$

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Verification is model checking: IMP $\models$ SPEC ?

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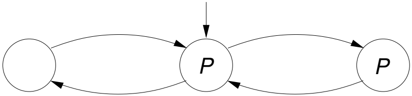

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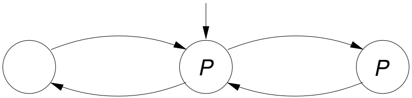

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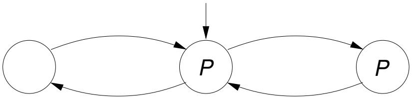

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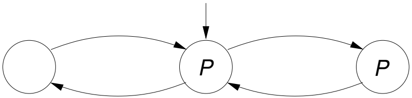

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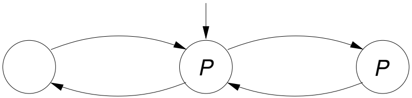

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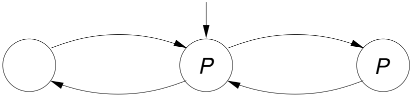

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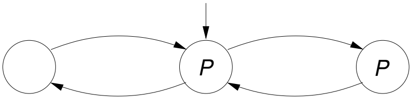

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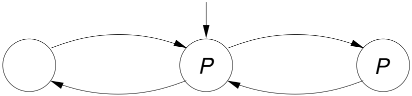

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- Finally, need to existentially quantify Q out:

$$\exists Q\ (Q \textit{ holds precisely at even positions } \textbf{and } \Box\,(Q \rightarrow P))$$

Monadic Second-Order Logic

Monadic Second-Order Logic (MSO($<$))

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Theorem (Büchi 1960)

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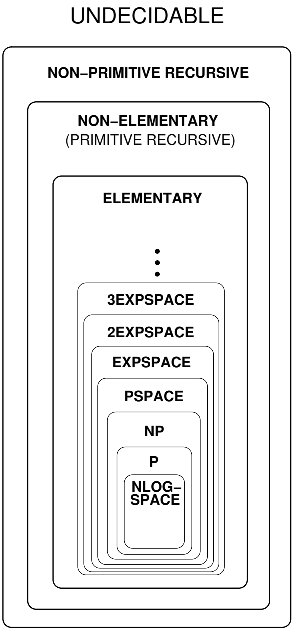
Corollary (Church 1960)

The model-checking problem for automata against MSO($<$) *specifications is decidable:*

$$M \models \varphi \quad \textit{iff} \quad L(M) \cap L(A_{\neg\varphi}) = \emptyset$$

Complexity


UNDECIDABLE
NON–PRIMITIVE RECURSIVE
NON–ELEMENTARY
(PRIMITIVE RECURSIVE)
ELEMENTARY
3EXPSPACE
2EXPSPACE
EXPSPACE
PSPACE
NP
P
NLOG–SPACE

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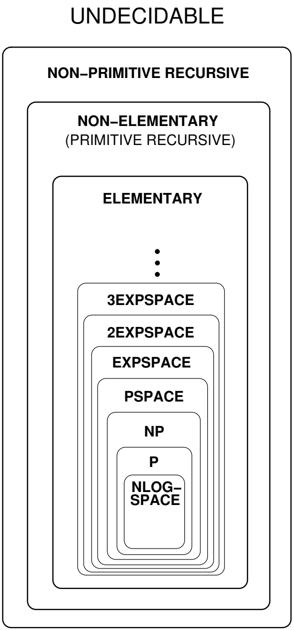

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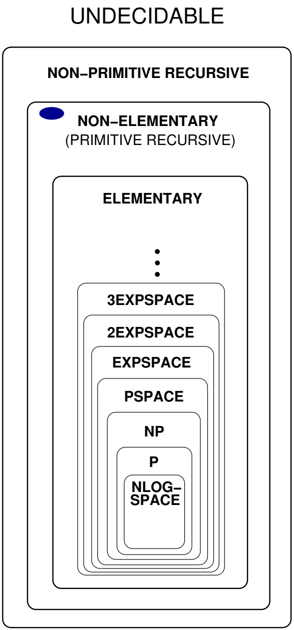

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**Theorem (Kamp 1968;
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LTL *and* FO($<$) *have precisely the same expressive power.*

But amazingly:

Theorem (Sistla & Clarke 1982)

LTL *satisfiability and model checking are PSPACE-complete.*

Logics and Automata

"The paradigmatic idea of the automata-theoretic approach to verification is that we can compile high-level logical specifications into an equivalent low-level finite-state formalism."

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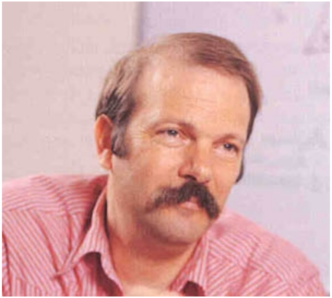

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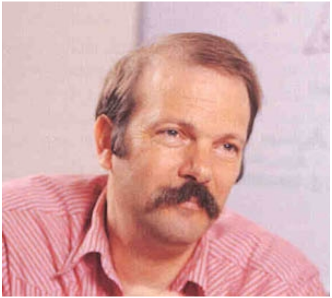

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Theorem

Automata are closed under all Boolean operations. Moreover, the language inclusion problem ($L(A) \subseteq L(B)$?) is PSPACE-complete.

The Classical Theory: Expressiveness

$\boxed{\mathrm{FO}(<)}$ - - - - - - - - - - - - - - - - - - $\boxed{\mathrm{LTL}}$

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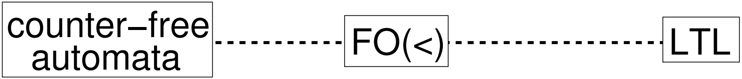

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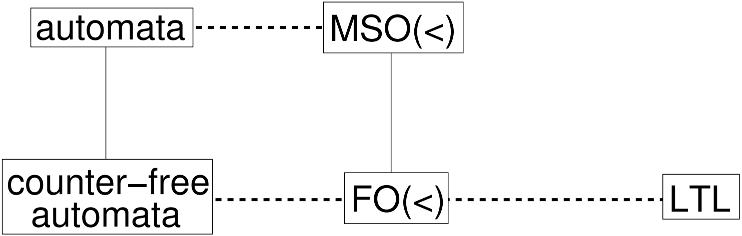

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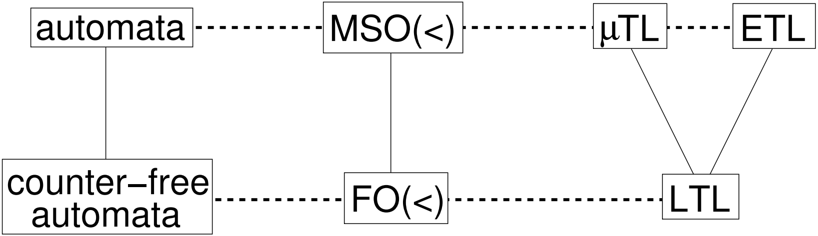

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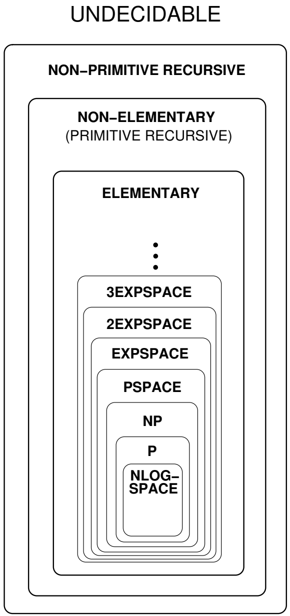

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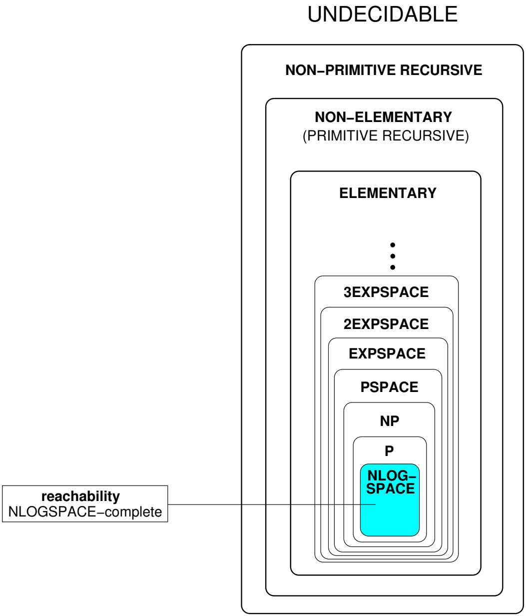

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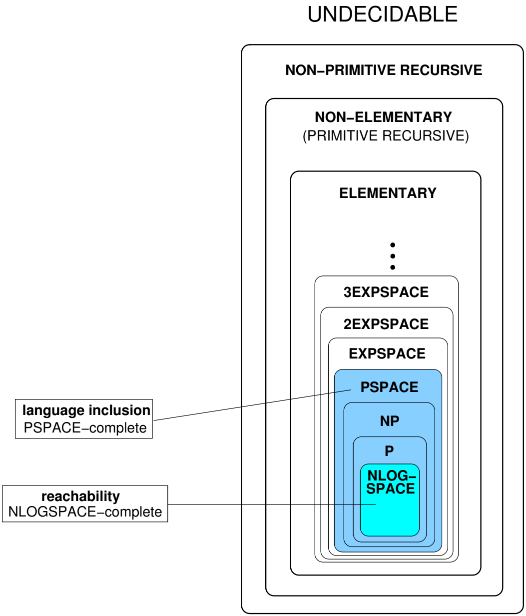

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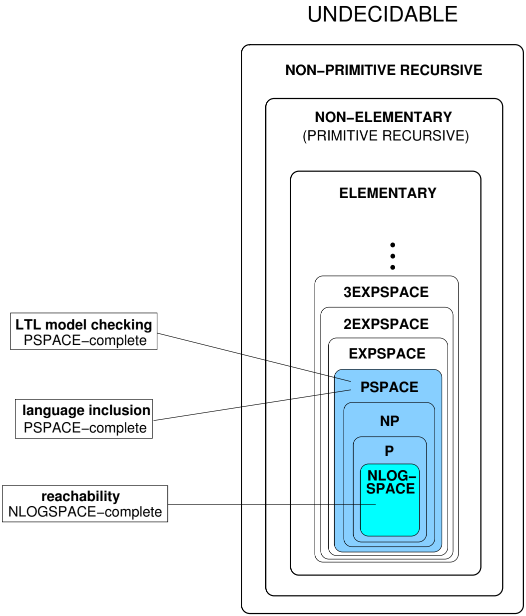

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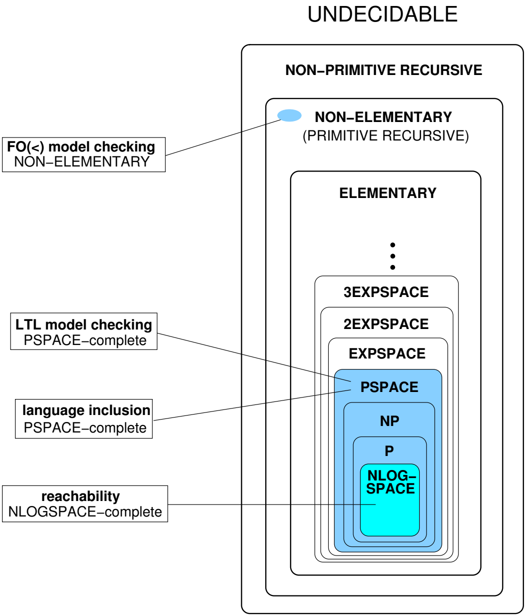

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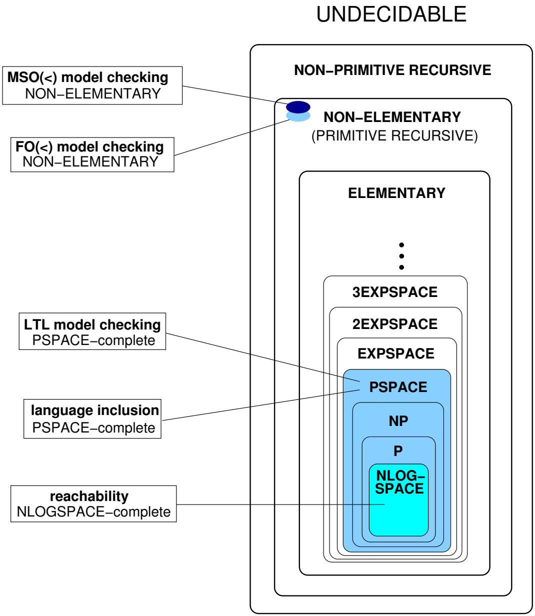


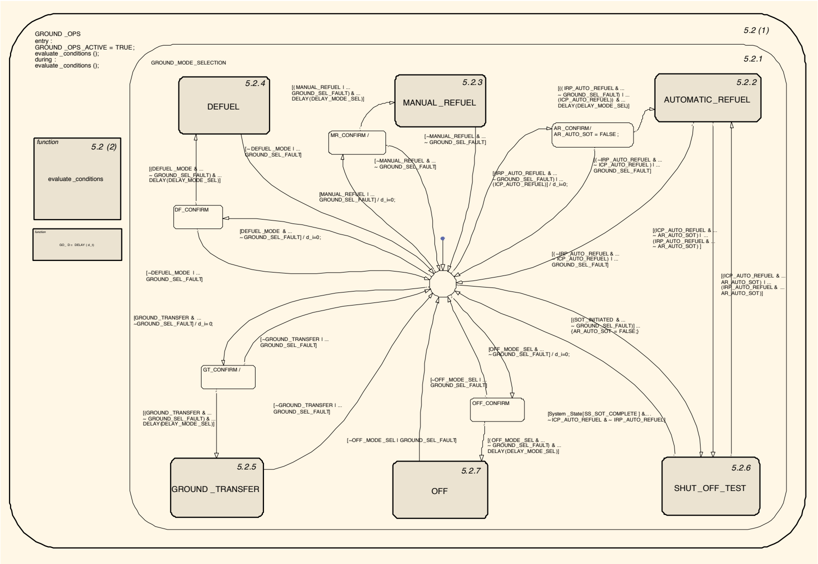
From Qualitative to Quantitative

"Lift the classical theory to the real-time world."

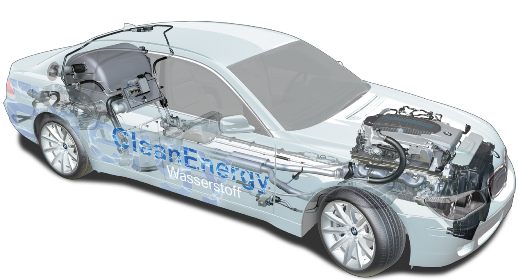
Boris Trakhtenbrot, LICS 1995

Airbus A350 XWB

A350 XWB Fuel Management Sub-System



BMW Hydrogen 7



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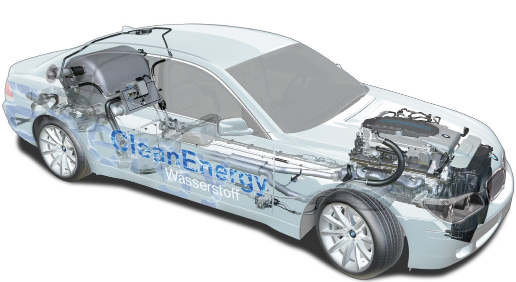
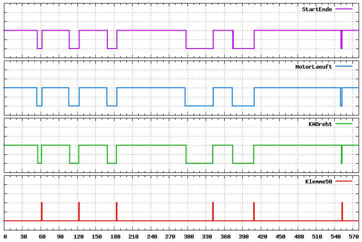
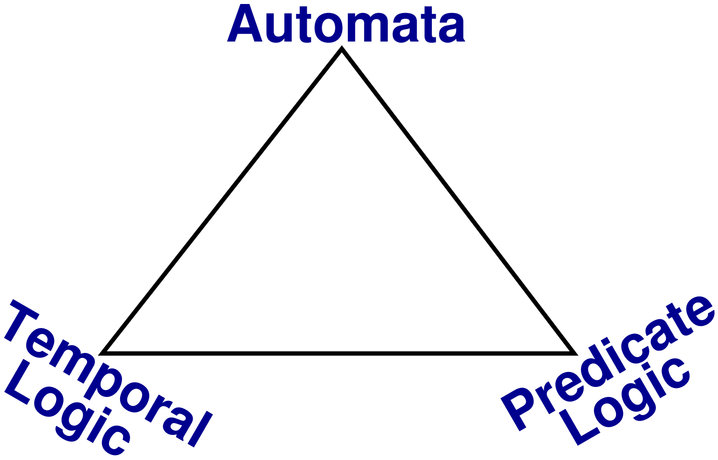




Timed Systems

Timed systems are everywhere…

- Hardware circuits
- Communication protocols
- Cell phones
- Plant controllers
- Aircraft navigation systems
- Sensor networks
- …


Automata
Temporal Logic
Predicate Logic

Timed Automata

Timed automata were introduced by Rajeev Alur at Stanford during his PhD thesis under David Dill:

- Rajeev Alur, David L. Dill: *Automata For Modeling Real-Time Systems*. ICALP 1990: 322-335
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$\Rightarrow$ Led to inaugural CAV Award (2008) *and* inaugural Church Award (2016)!

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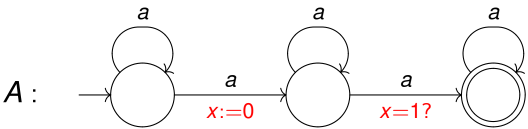
Unfortunately:

Theorem (Alur & Dill 1990)

Language inclusion is undecidable for timed automata.

An Uncomplementable Timed Automaton



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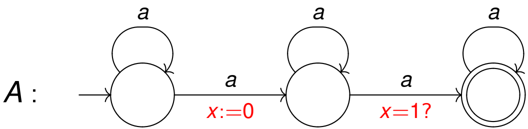

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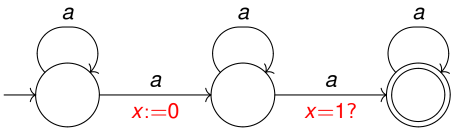
A :



$L(A)$:

An Uncomplementable Timed Automaton

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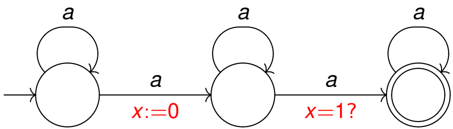

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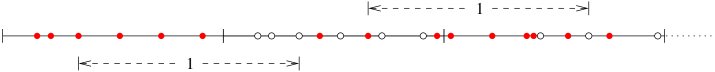

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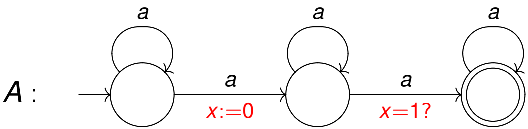

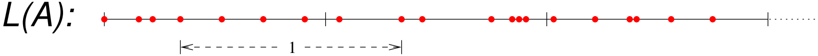

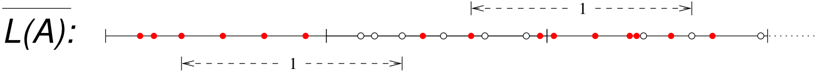

A cannot be complemented:
There is no timed automaton *B* with $L(B) = \overline{L(A)}$.

Metric Temporal Logic

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- MTL = LTL + timing constraints on operators:

$$\Box(\mathit{PEDAL} \rightarrow \Diamond_{[5,10]}\, \mathit{BRAKE})$$

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MTL *satisfiability and model checking are undecidable over* $\mathbb{R}_{\geq 0}$. *(Decidable but non-primitive recursive under certain semantic restrictions [O. & Worrell 2005].)*

Metric Predicate Logic

The first-order metric logic of order (FO($<$,$+1$)) extends FO($<$) by the unary function '$+1$'.

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For example, $\Box(\mathit{PEDAL} \rightarrow \Diamond_{[5,10]} \mathit{BRAKE})$ becomes

$$\forall x\, (\mathit{PEDAL}(x) \rightarrow \exists y\, (x + 5 \leq y \leq x + 10 \wedge \mathit{BRAKE}(y)))$$

Theorem (Hirshfeld & Rabinovich 2007)

FO($<, +1$) *is strictly more expressive than* MTL *over* $\mathbb{R}_{\geq 0}$.

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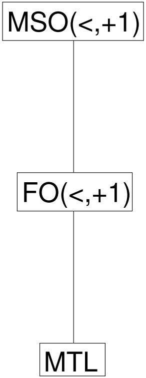
Corollary: FO$(<, +1)$, FO$(<, +\mathbb{Q})$, MSO$(<, +1)$, MSO$(<, +\mathbb{Q})$ satisfiability and model checking are all undecidable over $\mathbb{R}_{\geq 0}$.

The Real-Time Theory: Expressiveness



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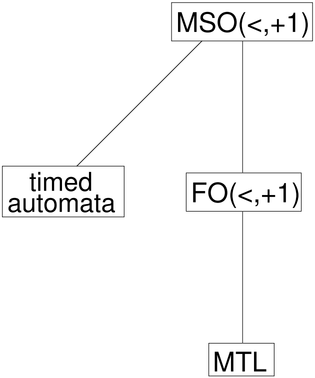
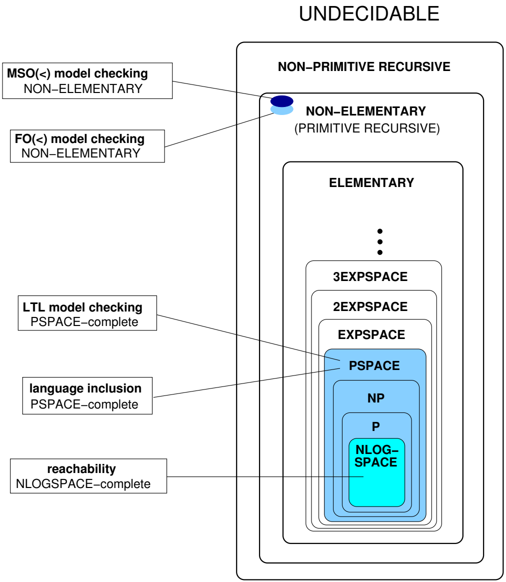


The Real-Time Theory: Complexity

Classical Theory

Real–Time Theory



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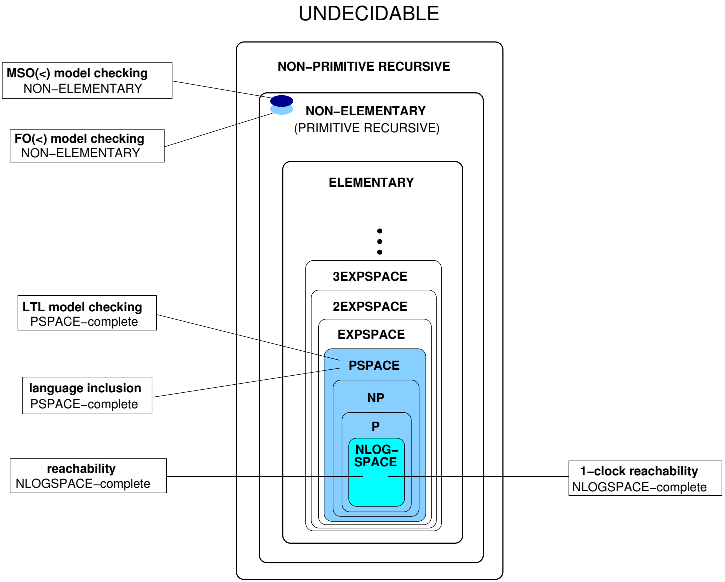

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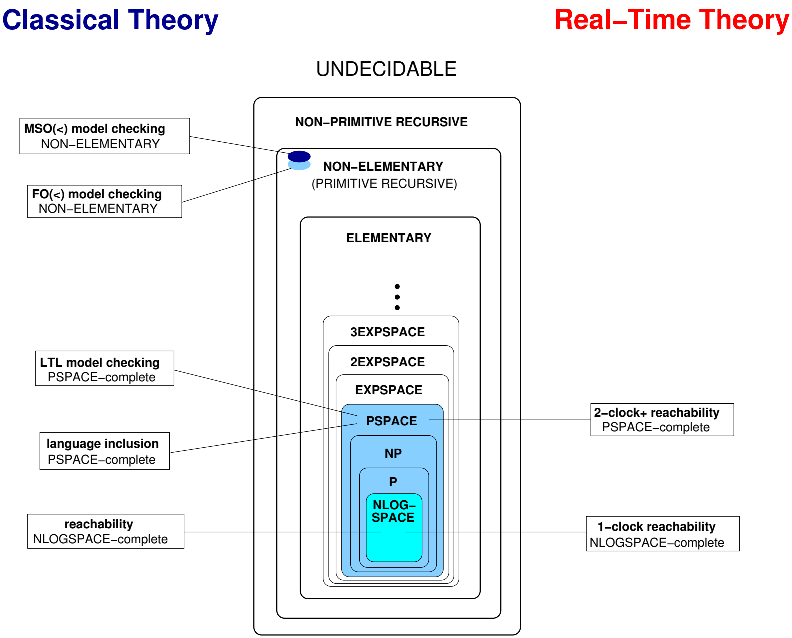

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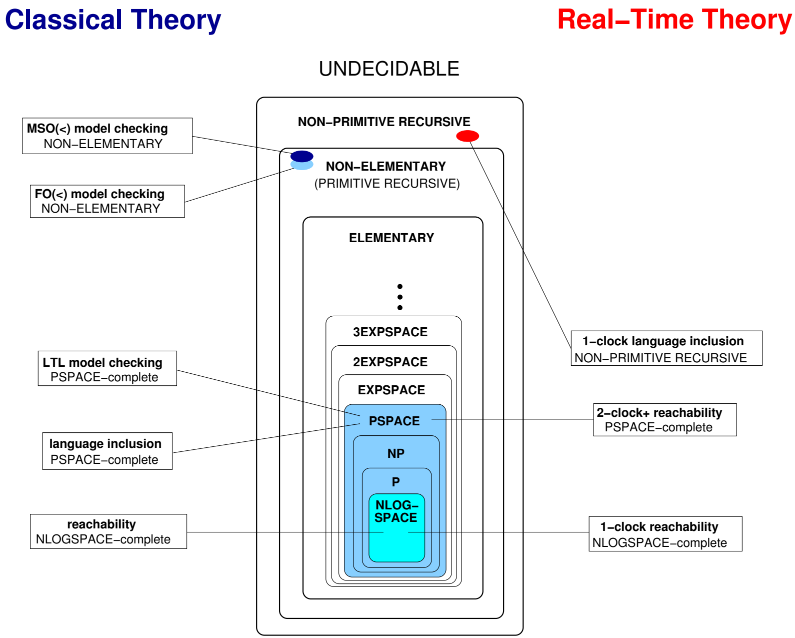

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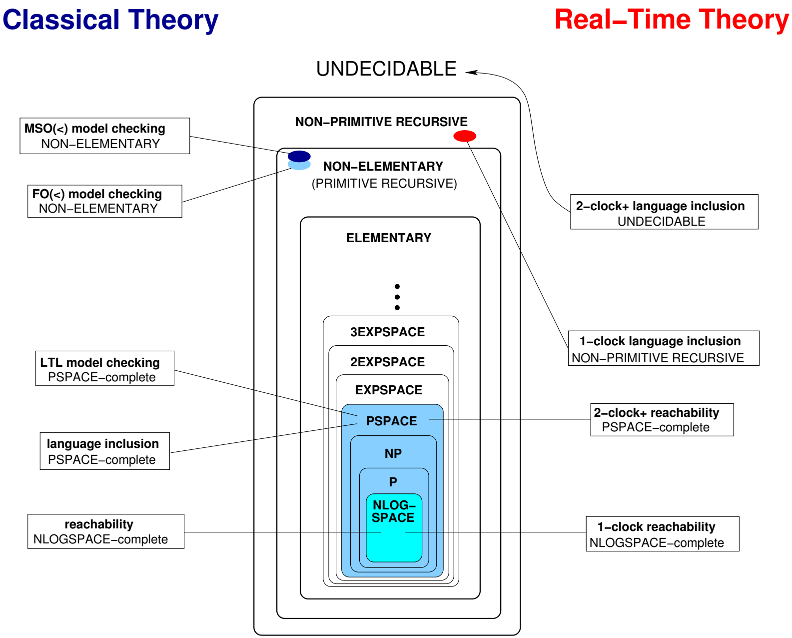

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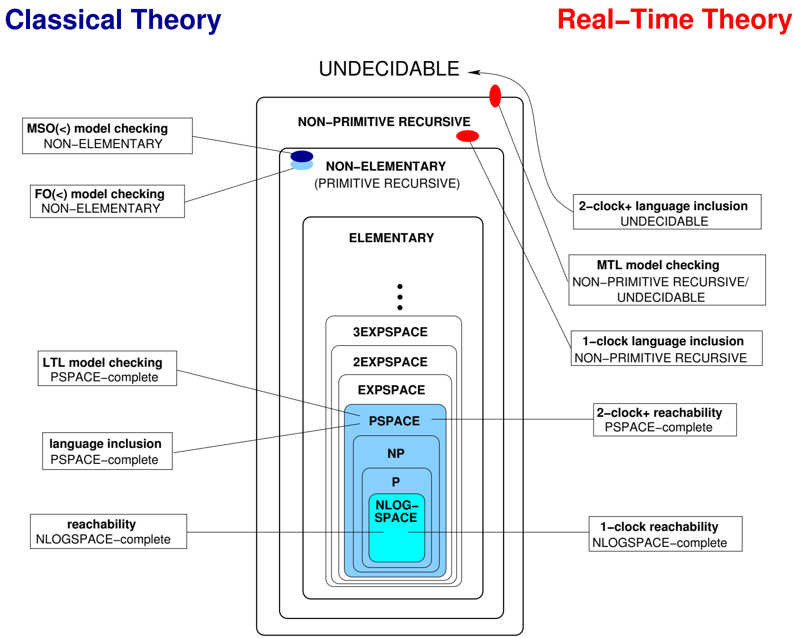

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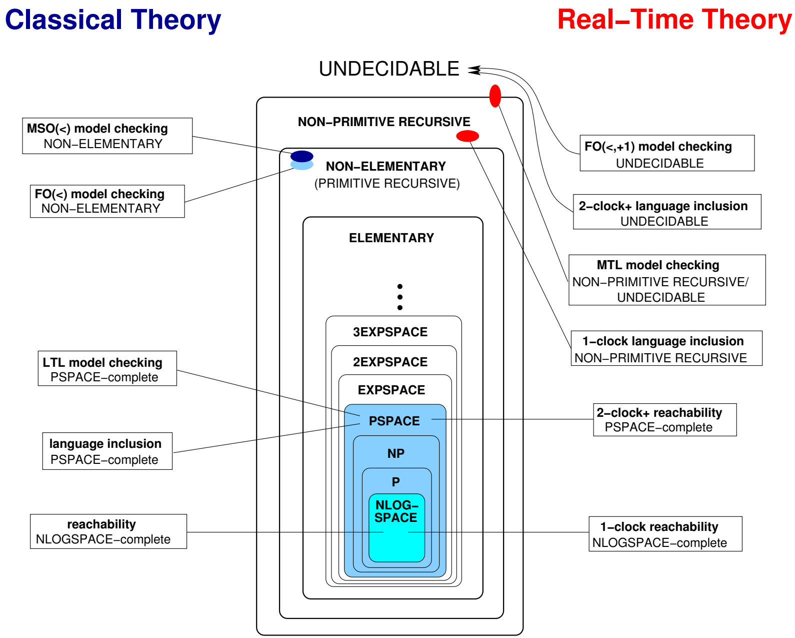

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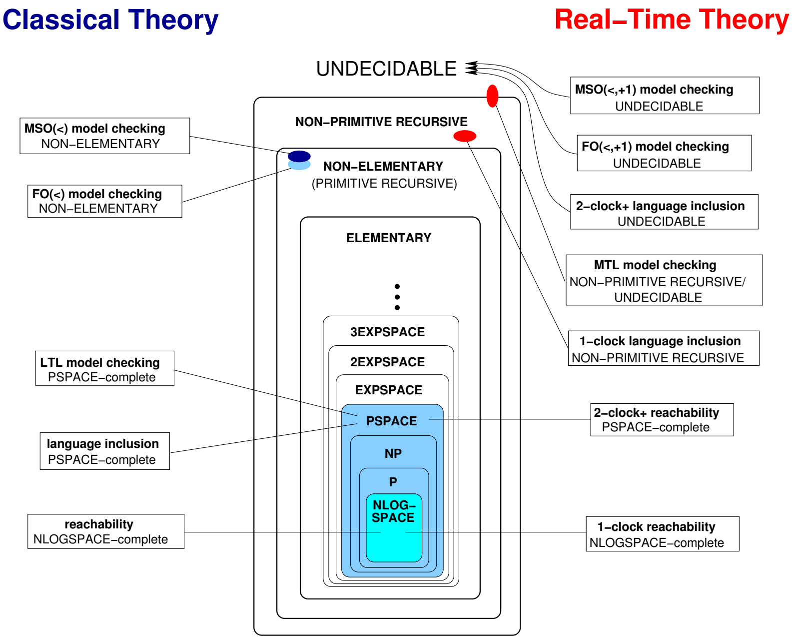

Key Stumbling Block

Theorem (Alur & Dill 1990)

Language inclusion is undecidable for timed automata.

Timed Language Inclusion: Some Related Work

- Topological restrictions and digitization techniques: [Henzinger, Manna, Pnueli 1992], [Bošnački 1999], [O. & Worrell 2003]
- Fuzzy semantics / noise-based techniques: [Maass & Orponen 1996], [Gupta, Henzinger, Jagadeesan 1997], [Fränzle 1999], [Henzinger & Raskin 2000], [Puri 2000], [Asarin & Bouajjani 2001], [O. & Worrell 2003], [Alur, La Torre, Madhusudan 2005]
- Determinisable subclasses of timed automata: [Alur & Henzinger 1992], [Alur, Fix, Henzinger 1994], [Wilke 1996], [Raskin 1999]
- Timed simulation relations and homomorphisms: [Lynch *et al.* 1992], [Taşiran *et al.* 1996], [Kaynar, Lynch, Segala, Vaandrager 2003]
- Restrictions on the number of clocks: [O. & Worrell 2004], [Emmi & Majumdar 2006]

Time-Bounded Language Inclusion

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Instance: Timed automata A, B, and time bound $T \in \mathbb{N}$

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- Inspired by Bounded Model Checking.
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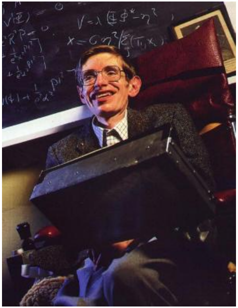
- Universe's lifetime is believed to be bounded anyway…



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- Unfortunately, timed automata cannot be complemented even over bounded time...
- Key to solution is to translate problem into logic: Behaviours of timed automata can be captured in MSO($<$,$+1$)
- This reverses Vardi's 'automata-theoretic approach to verification' paradigm!

Monadic Second-Order Logic

Theorem (Shelah 1975)

MSO($<$) *is undecidable over* $[0,1)$.

Monadic Second-Order Logic

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By contrast,

Theorem

- MSO($<$) *is decidable over* $\mathbb{N}$ *[Büchi 1960]*
- MSO($<$) *is decidable over* $\mathbb{Q}$, *via [Rabin 1969]*

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Timed behaviours are modelled as flows (or signals):

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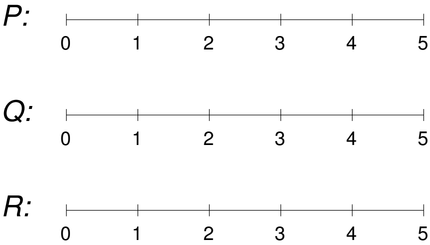

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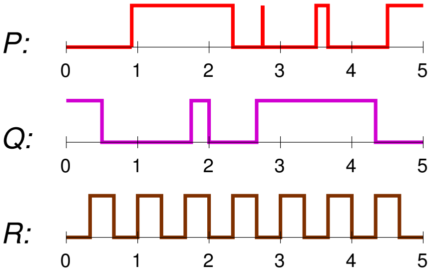

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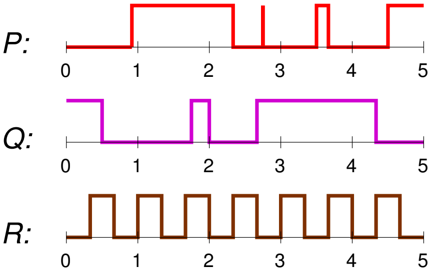

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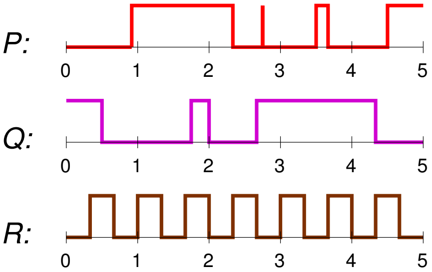

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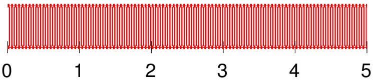

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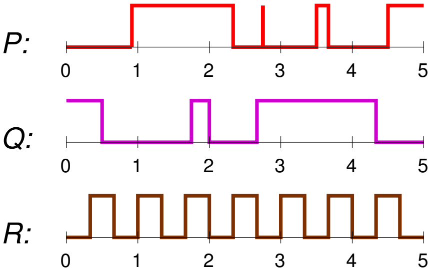

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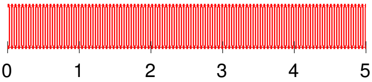

Then:

Theorem (Rabinovich 2002)

MSO($<$) *satisfiability over finitely-variable flows is decidable.*

The Time-Bounded Theory of Verification

Theorem

For any bounded time domain $[0, T)$, satisfiability and model checking are decidable as follows:

MSO$(<, +1)$	*NON-ELEMENTARY*
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MTL *and* FO$(<, +1)$ *are equally expressive over any fixed bounded time domain* $[0, T)$*.*

Theorem

Given timed automata A, B, and time bound $T \in \mathbb{N}$, the time-bounded language inclusion problem $L_T(A) \subseteq L_T(B)$ is decidable and 2EXPSPACE-complete.

MSO($<$,+1) Time-Bounded Satisfiability

Key idea: eliminate the metric by ‘vertical stacking’.

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- Conclude by invoking decidability of MSO($<$).

From MSO($<$,$+1$) to MSO($<$)

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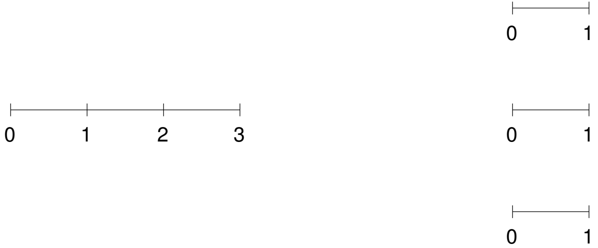

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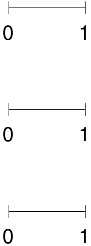

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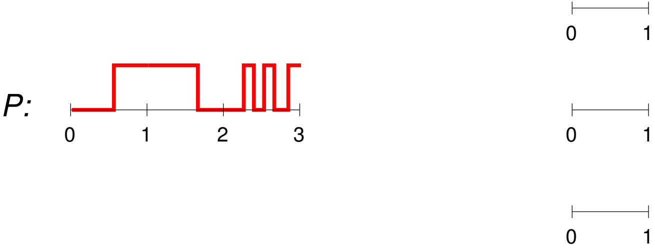

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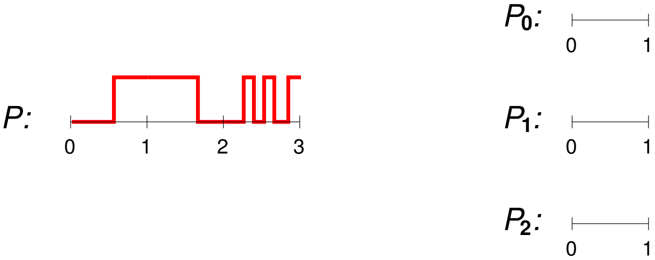

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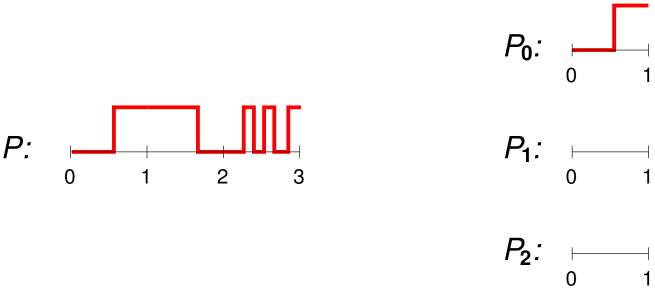

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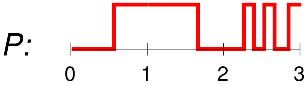

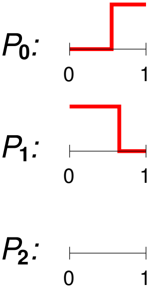

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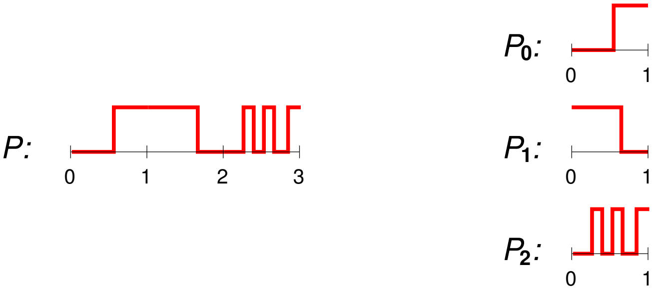

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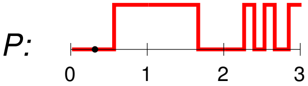

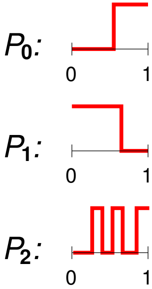

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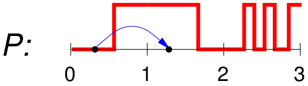

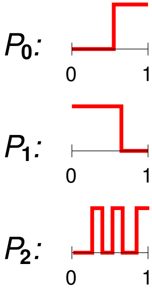

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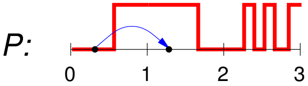

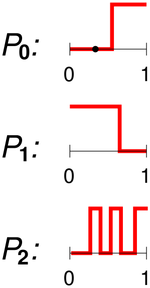

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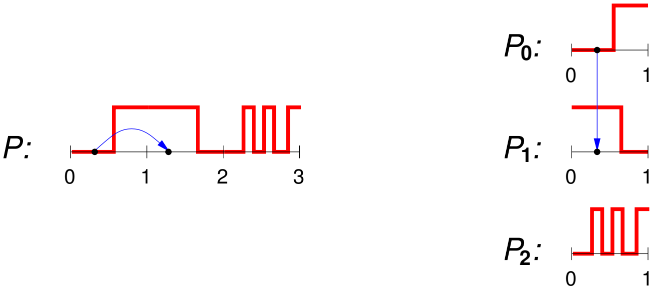

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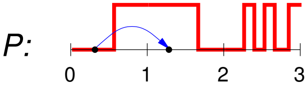

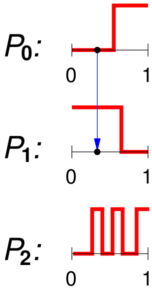

Replace every:

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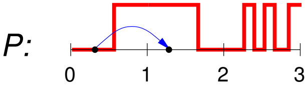

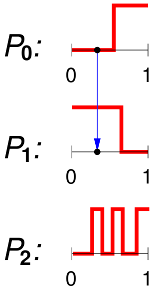

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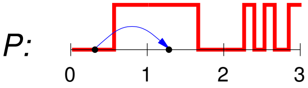

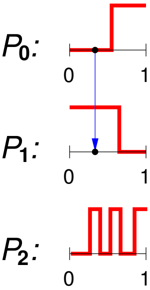

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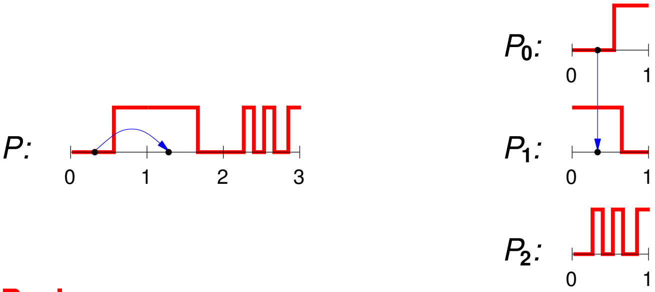

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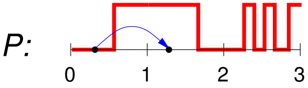

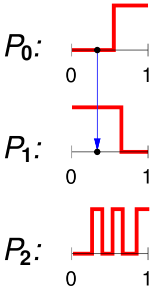

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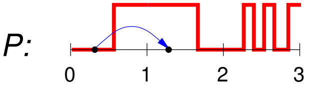
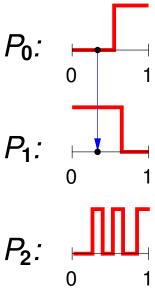
- $P(x+k)$

From MSO($<$,$+1$) to MSO($<$)

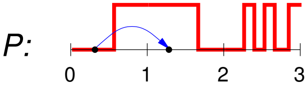


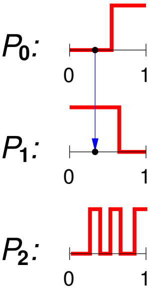


Replace every:

- $\forall x\, \psi(x)$ **by** $\forall x\ (\psi(x) \wedge \psi(x+1) \wedge \psi(x+2))$
- $x + k_1 < y + k_2$ **by** $\begin{cases} x < y & \text{if } k_1 = k_2 \\ \textbf{true} & \text{if } k_1 < k_2 \\ \textbf{false} & \text{if } k_1 > k_2 \end{cases}$
- $P(x+k)$ **by** $P_k(x)$

From MSO(<,+1) to MSO(<)





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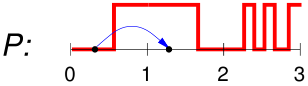
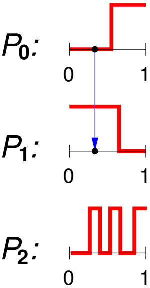
- $\forall P\, \psi$

From MSO($<$,$+1$) to MSO($<$)





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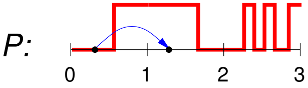
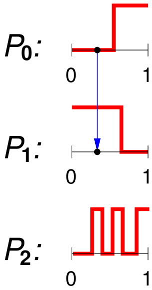
- $P(x+k)$ **by** $P_k(x)$
- $\forall P\, \psi$ **by** $\forall P_0\, \forall P_1\, \forall P_2\, \psi$

From MSO(<,+1) to MSO(<)





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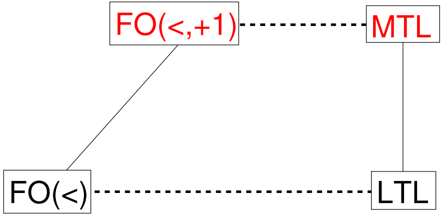
- $P(x+k)$ **by** $P_k(x)$
- $\forall P\, \psi$ **by** $\forall P_0\, \forall P_1\, \forall P_2\, \psi$

Then φ is satisfiable over $[0, T)$ $\iff$ $\overline{\varphi}$ is satisfiable over $[0, 1)$.

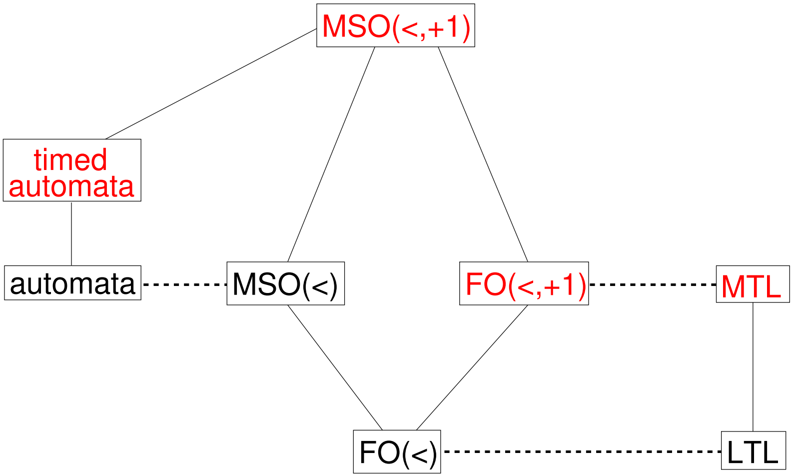
The Time-Bounded Theory: Expressiveness

$\boxed{\text{FO}(<)}$ - - - - - - - - - - $\boxed{\text{LTL}}$

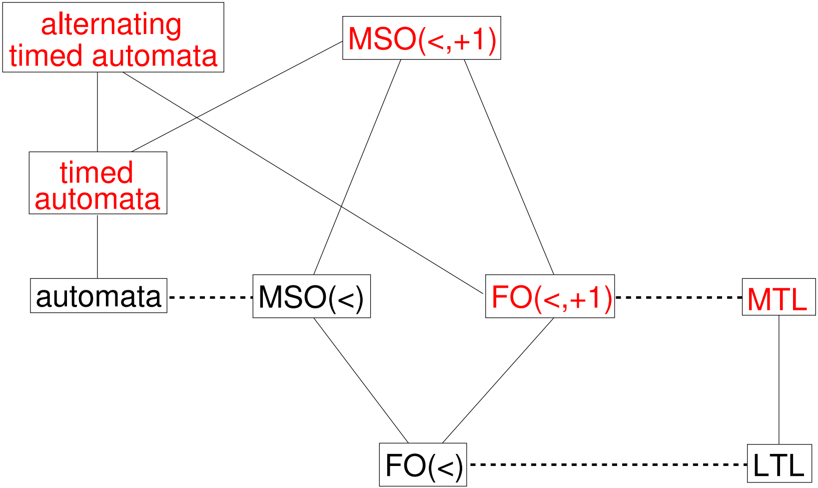
The Time-Bounded Theory: Expressiveness



The Time-Bounded Theory: Expressiveness



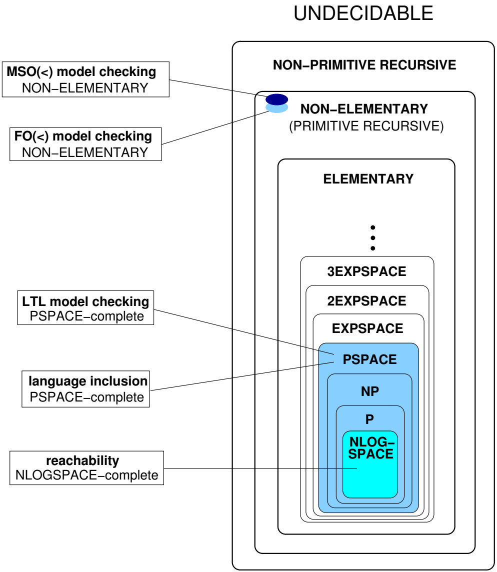
The Time-Bounded Theory: Expressiveness



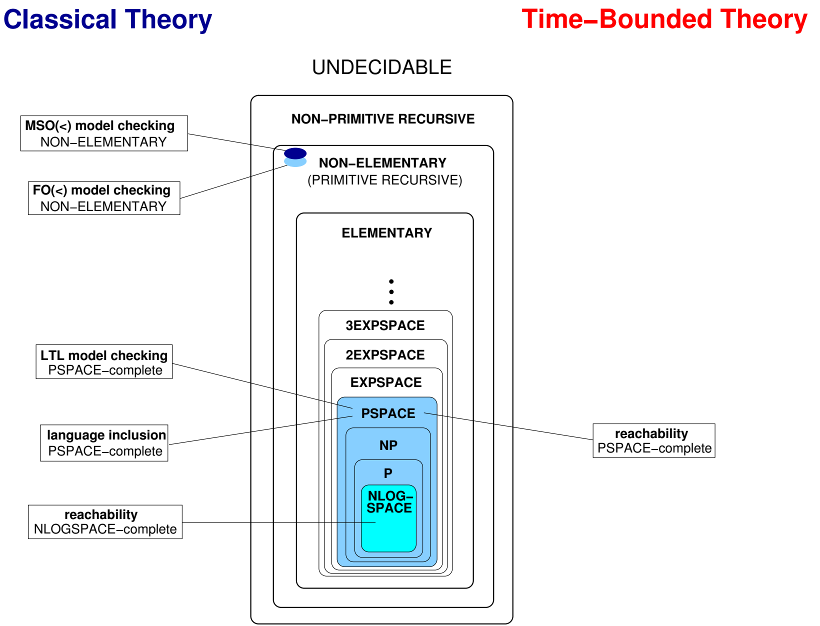
The Time-Bounded Theory: Complexity

Classical Theory

Time–Bounded Theory



The Time-Bounded Theory: Complexity



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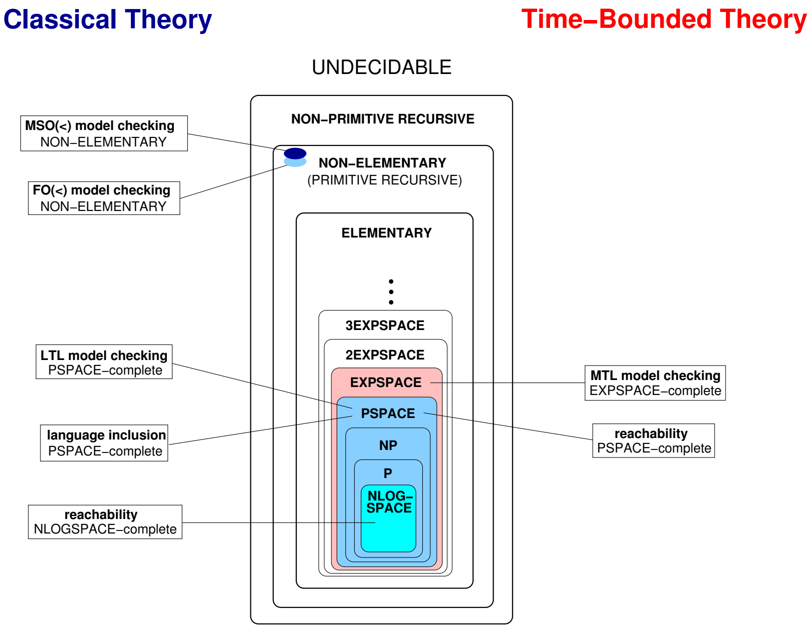

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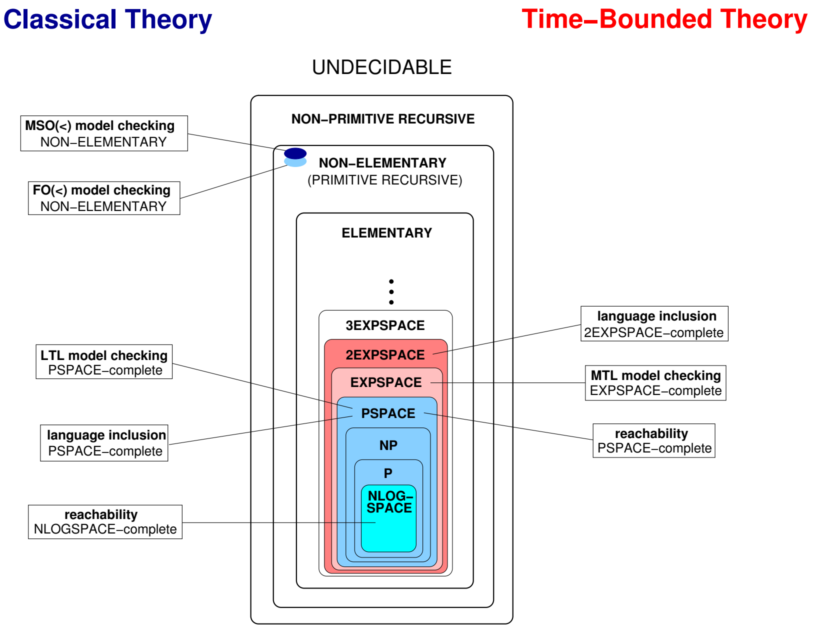

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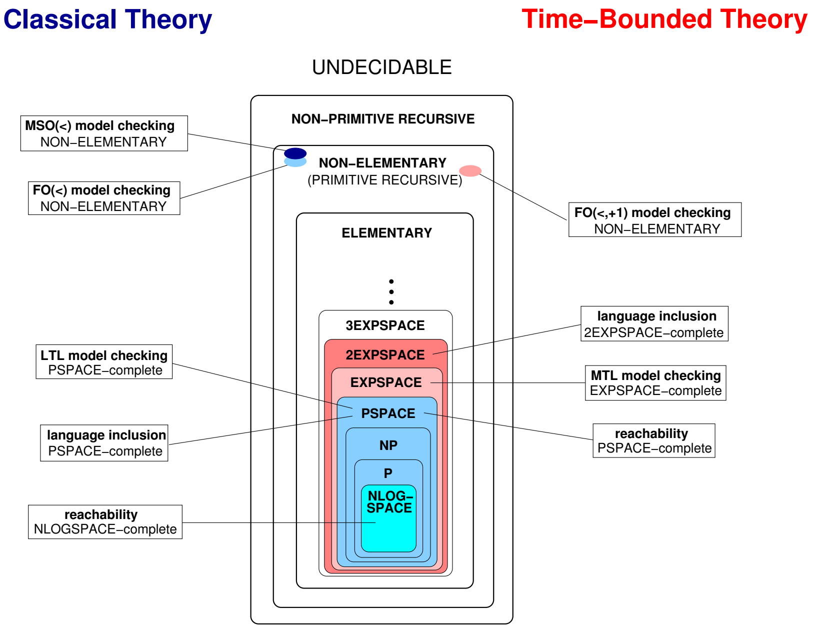

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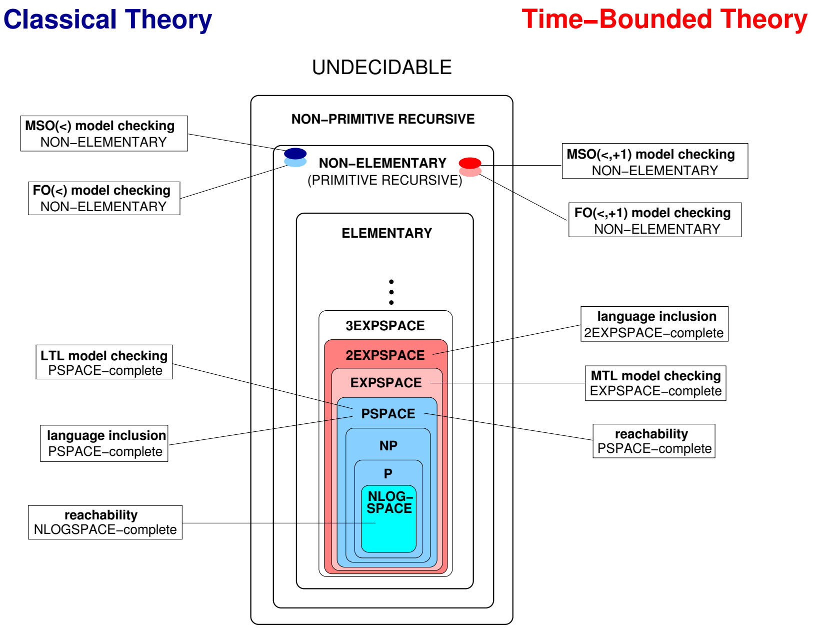

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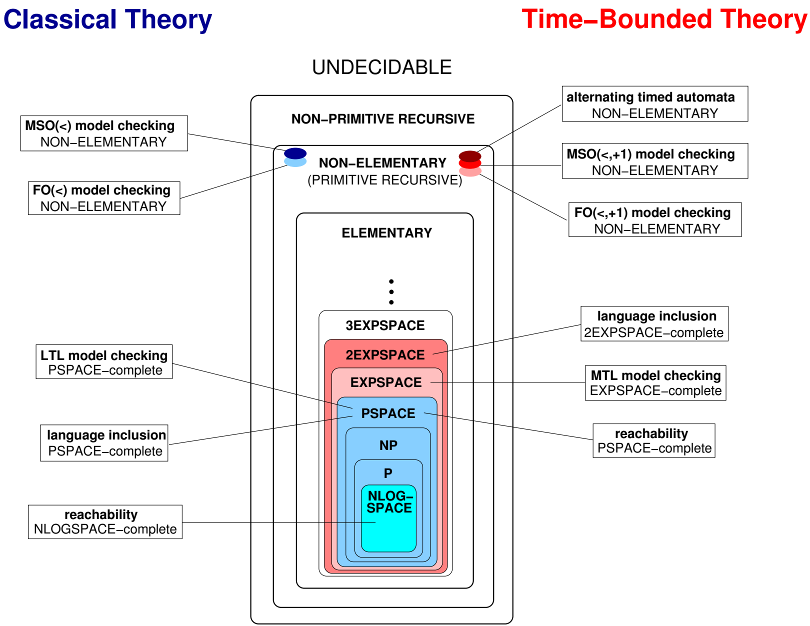

Conclusion and Perspective

- For real-time systems, the time-bounded theory is much better behaved than the real-time theory.

Conclusion and Perspective

- For real-time systems, the time-bounded theory is much better behaved than the real-time theory.

Going forward:

- Extend the theory further!
  - Branching-time
  - Timed games and synthesis
  - Weighted and hybrid automata
  - ...
- Algorithmic and complexity issues
- Expressiveness issues
- Implementation and case studies