Compact routing, spanners, multipath routing

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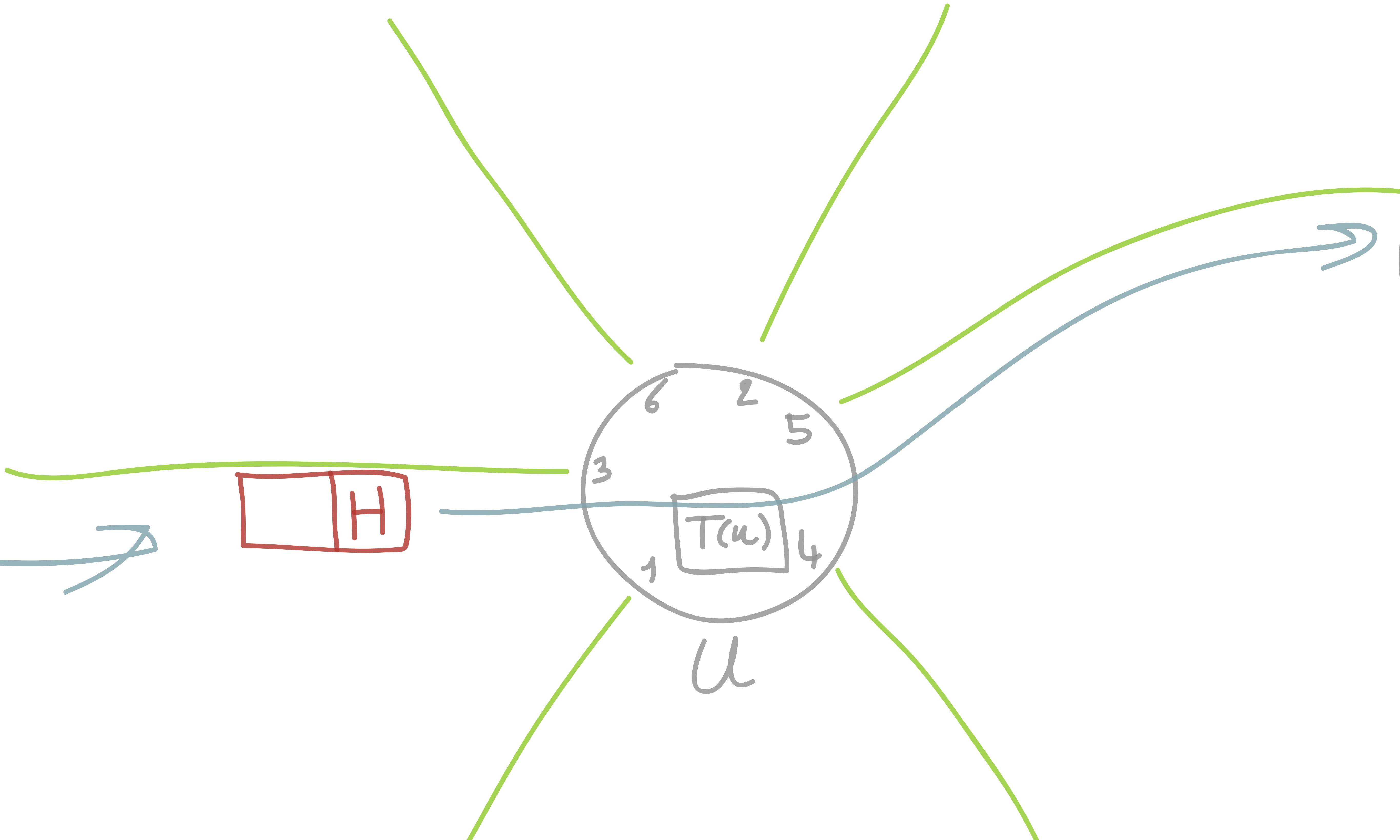
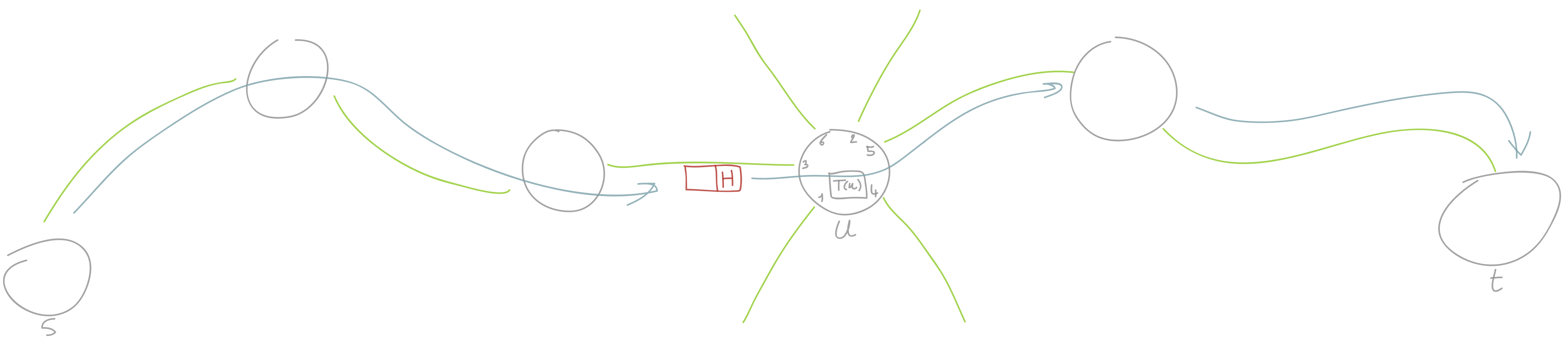
Compact routing

- Information theory approach to network routing
- Trade-off space vs length :
  - routing table size
  - route length

Routing model



Routing model



Routing scheme

- Each node has a routing table $T(u)$
- Each node has a label $L(u)$
- Routing function f :
  - Packet P arrives at u with header H or $H := L(t)$
  - $(H', p) := f(T(u), H)$
  - Forward P on port p with header H'

Route

- $Route(s \to t)$: $s=u_0 u_1,\ldots,u_{d-1} u_d=t$ with :
  - $H0=L(t)$
  - $\forall i$: $(H_{i+1}, p_{i+1}) = f(T(u_i), H_i)$
  - $\forall i$: $neighbor(u_i, p_{i+1}) = u_{i+1}$
  - $f(T(u_d), H_d) = 0$ (packet arrives)
- Route length is $|Route(s \to t)| = d$

Problem

- Given a network G define f and compute T, L s.t. :
- for all s,t∈V(G) : all packets from s to t arrive.
- Complexity measures :
  - Table size = max_u |T(u)|
  - Lable size = max_u |T(u)|
  - Stretch = max_st |Route(s → t)| / d_G(s,t)
  - Time complexity of f (usually O(1))
- Classical routing: u stores next hop for all t
  - size n log n and stretch 1

Variants

- Labelled : $L(u)$ is designer choice (DNS)
- Name independent : $L(u)$ is chosen by an adversary
- Designer port / fixed port
- Weighted : edges of G have weights
- Oblivious : $H_{i+1} = H_i$
- G is a tree, a planar graph, a clique, is doubling, . . .
- (For a survey : [Dom 07])

Main results

- [Kleinrock & Kamoun 77] Hierarchical routing
- [Peleg & Upfal 88] Trade-off size $O(n^{1+1/k})$ vs stretch $O(k)$ for integral $k \geq 1$
- [Gavoille & Pérennès 96] stretch$\leq 1 \Rightarrow$ size$\geq n \ln n$
- [Gavoille & Gengler 97] stretch$<3 \Rightarrow$ size$\geq n$
- [Awerbuch & al 89] name independent
- [TZ01 / Fraigniaud & G 02] trees$\Rightarrow$size$\leq (\ln n)^2/\ln\ln n$
- [Thorup & Zwick 01] size$<\sqrt{(n \ln n)}$, stretch≤ 3
- [Abraham & al 04] same but name independent

Open issues

- Applicable to Internet ? [Mahadevan & al 04]
- Destinations are not nodes.
- Need for distributed computation ?
- Consider throughput, delay, traffic rather than just size.

Link with spanners

- D
- L

Main open issue : additive stretch

- Every graph has a (2k-1)-spanner of size $O(n^{1+1/k})$
- (a,b)-spanner : for all u,v $d_H(u,v) \leq a\, d_G(u,v) + b$
- Has every graph a (1,2k-2)-spanner of small size ?
- Every graph has a (1,2)-spanner of size $O(n^{1+1/2})$ [Aingworth & al. 99]
- Every graph has a (1,6)-spanner of size $O(n^{1+1/3})$ [Baswana & al. 06]
- Every graph has a (1,4)-spanner of size $O(n^{1+2/3})$ [Chechik & al. 13]

Multipath

Multipath spanners

- Definition : H is a k-path s-spanner of G iff $H \subseteq G$ and $d_H^k(s,t) \leq d_G^k(s,t)$ for all s,t.
- Th [Gavoille, Godfroy & V. 11] : $\forall k \forall p \exists c_1 \exists c_2\ \forall G \exists H$, H is a k-path $c_1(1+k/p)^{2p-1}$-spanner of G with at most $c_2\, n^{1+1/p} \log^2 n$ edges.
- Th2 ($k=p=2$) : Every graph has a k-path spanner with stretch almost 2 and $O(n^{1+1/2})$ edges.
- Conjecture : Every graph has a k-path spanner with stretch $O(kp)$ and $O(n^{1+1/p})$ edges.

Multipath routing

- Work in progress [Mathieu, de Montgolfier & V.]
- Classical routing : T(u)={NextHop(t) for all t}
- Problem : define for all u,s,t,i a neighbor NextHop_u(s,t,i) such that forwarding along index i=1..k results in disjoint routes.
- Brute force : size kn^2
- Can you factorize paths ? (as in the case k=1)
- If so (same NextHop(t,i) $\forall$s) : size kn
- Conjecture : G k-connected $\Rightarrow$ yes you can

Vote

- G k-connected $\Rightarrow$ you can factorize NextHop
- Don't worry, I don't know the answer.
- Hint 1 : if you want shortest k-path then No.
- Hint 2 : if $k = 2$ then Yes.
- Thank you, we will try to prove . . . first.