UNIVERSITY OF OXFORD

DEPARTMENT OF COMPUTER SCIENCE

Games with bound guess actions

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27 April 2016

joint work with
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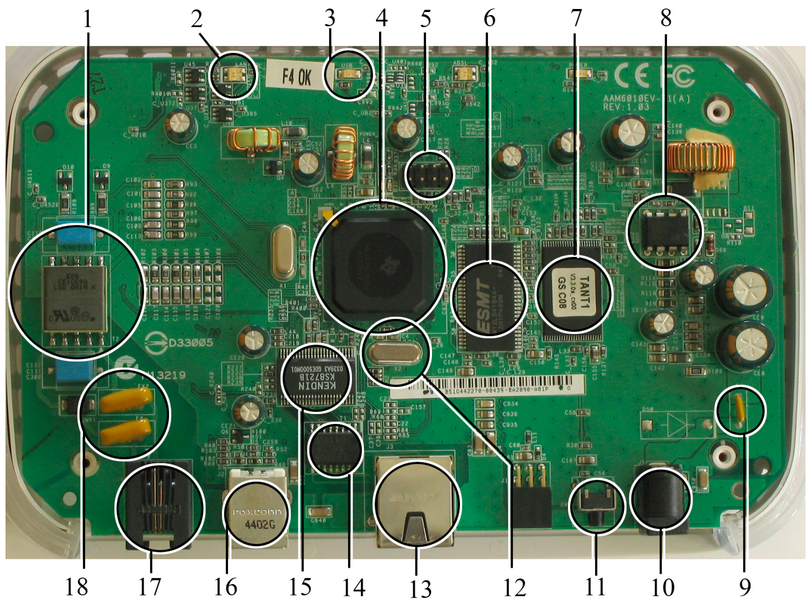
(at LICS'16)

INSTITUT DE RECHERCHE EN INFORMATIQUE FONDAMENTALE

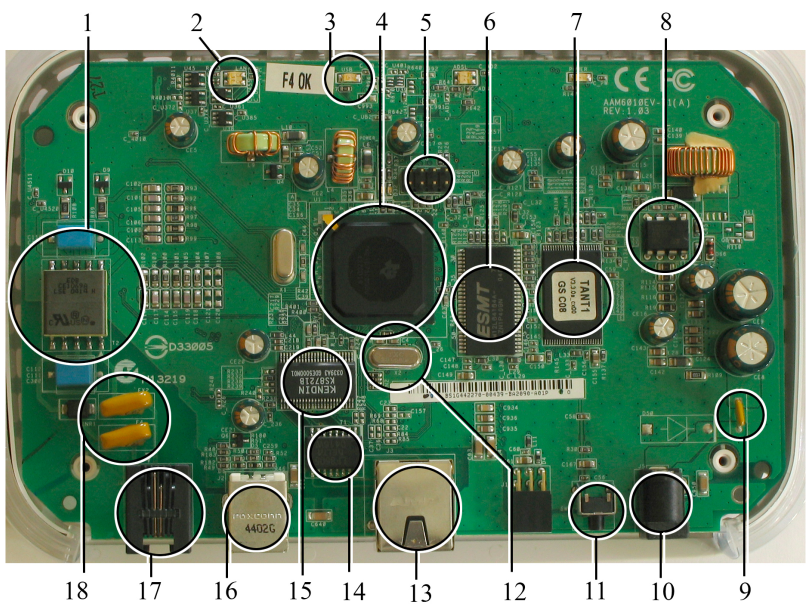
cnrs

Games for model checking



A system

Games for model checking



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A specification that
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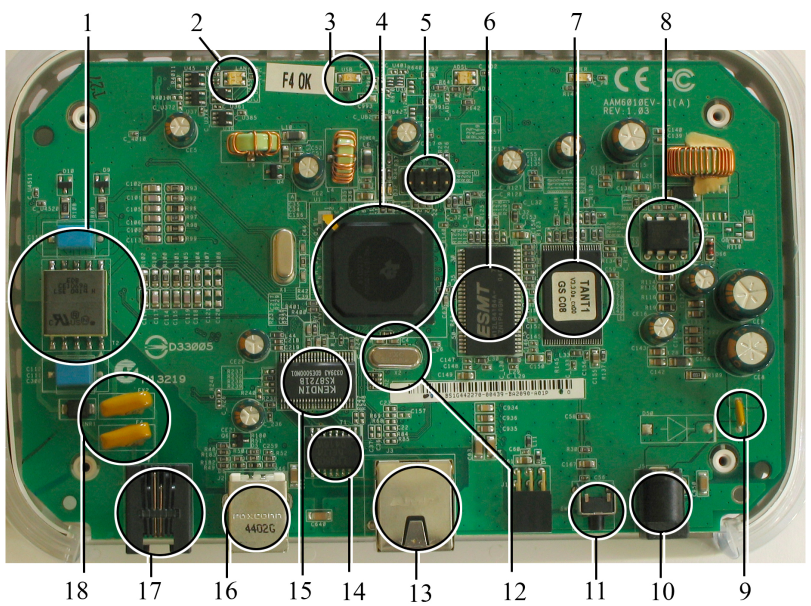

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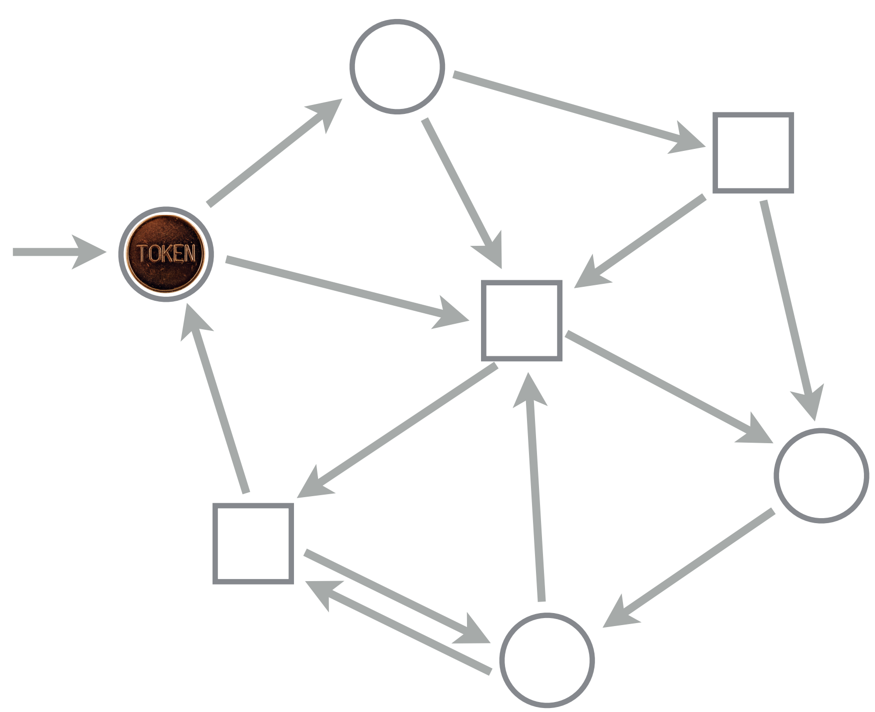

A game involving
- a prover
- a falsifier

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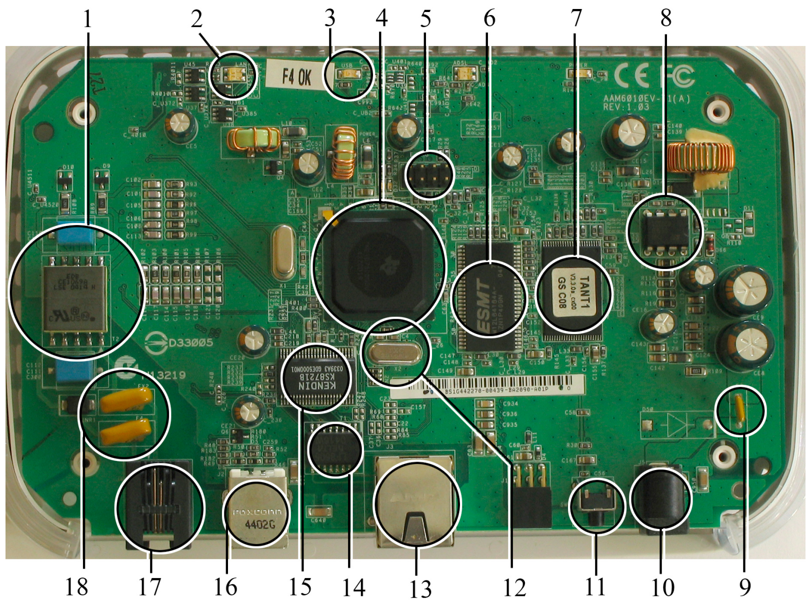

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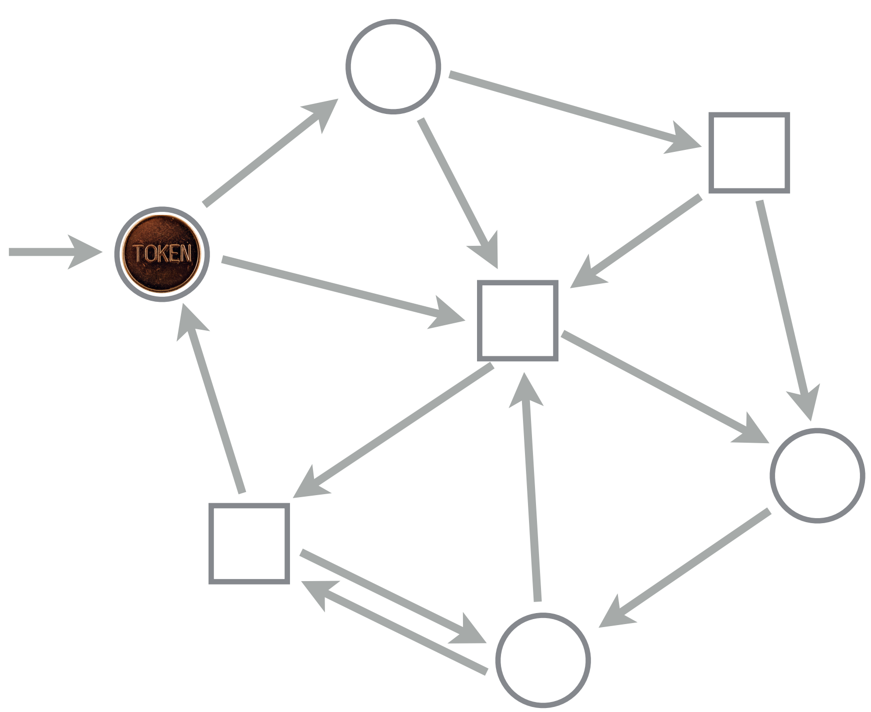
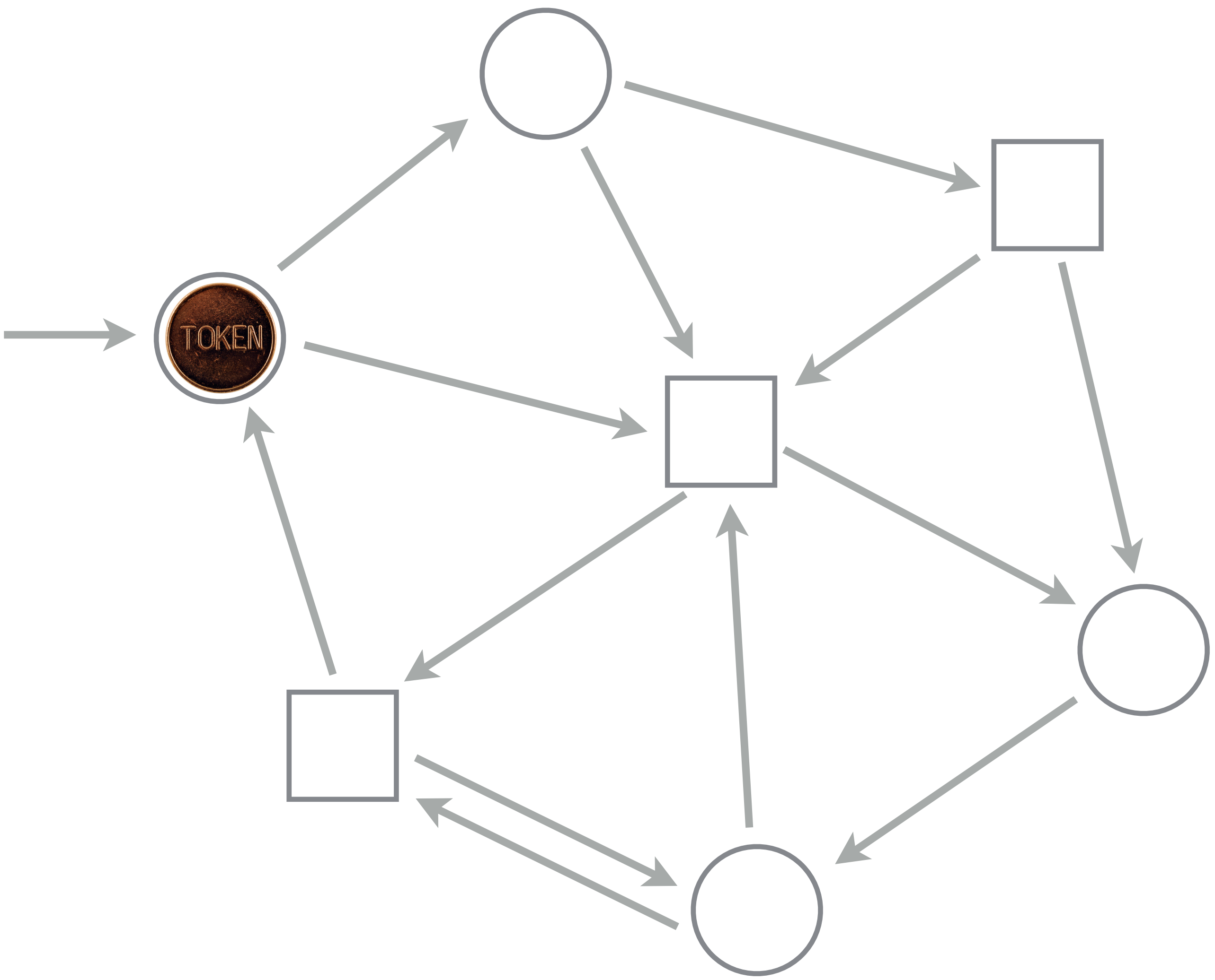


A game involving
- a prover
- a falsifier

such that prover can win if and only if the system satisfies the specification.

Games

(Two players, antagonistic, turn-based)



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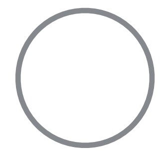

the existential player
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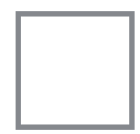

the universal player
= the environment = falsifier.

A unique token is placed, and is controlled by the owner of the vertex, choosing the transition to follow.

The winner is determined based on the infinite sequence of moves.

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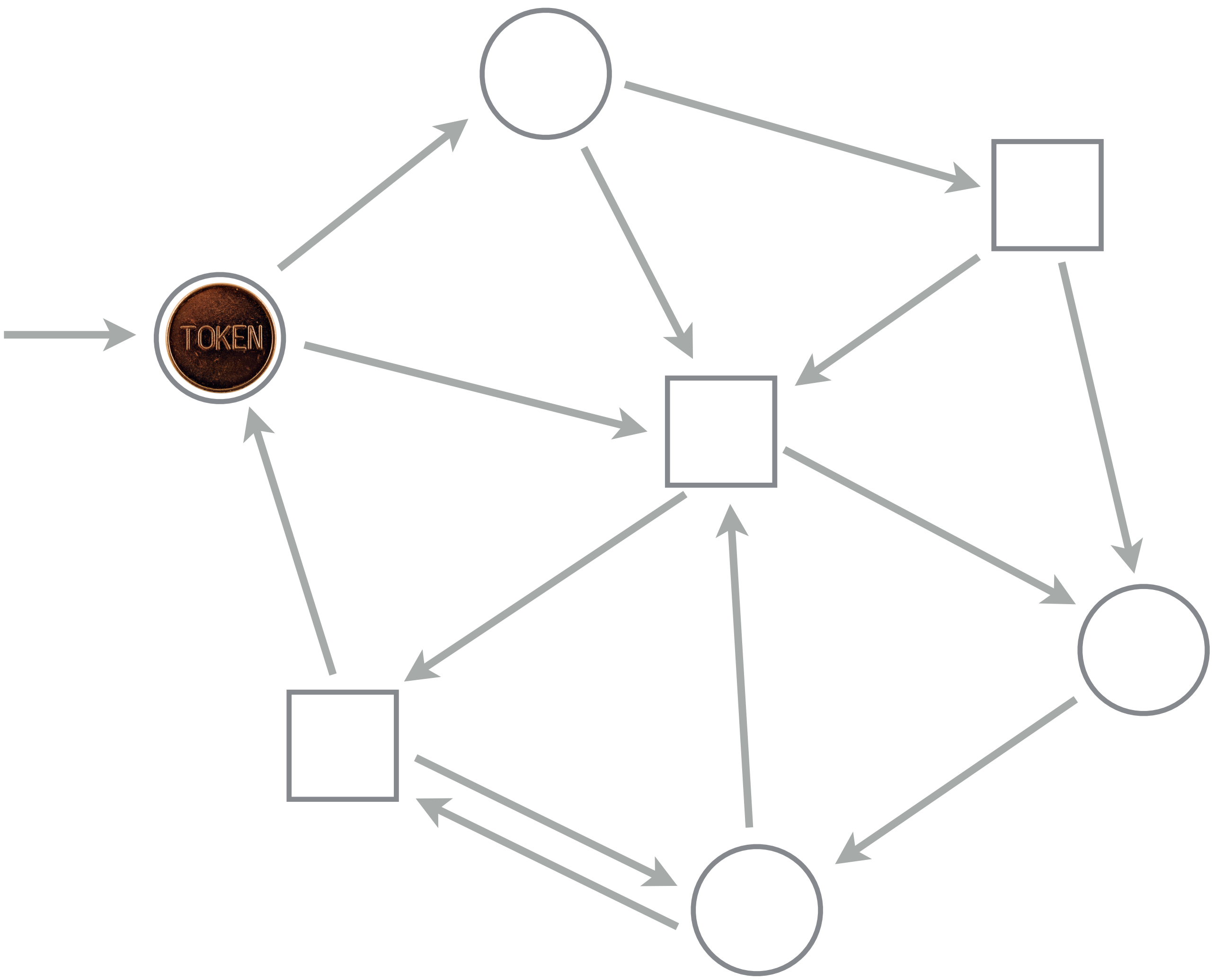

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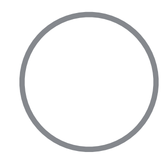

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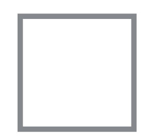

the **universal player** = the environment = falsifier.

A unique **token** is placed, and is controlled by the owner of the vertex, choosing the transition to follow.

The **winner** is determined based on the infinite sequence of moves.

Usually, moves are labelled by actions, and a (regular) set of winning sequences of actions is fixed.

Games with bound guess actions

Idea: players can play numbers (non-negative integers), which are promises on the evolution of some quantity.

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« every request is treated »

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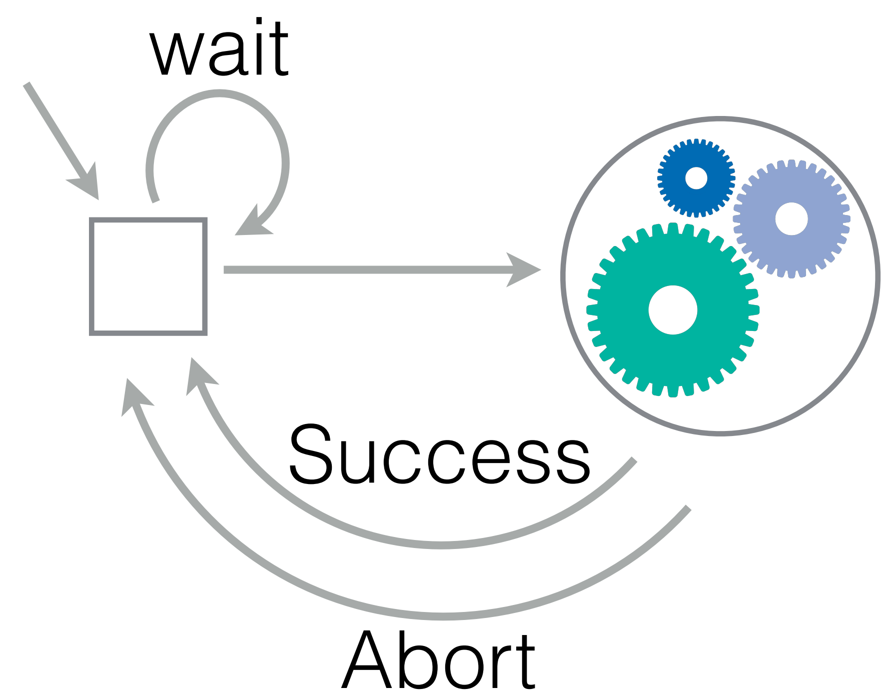

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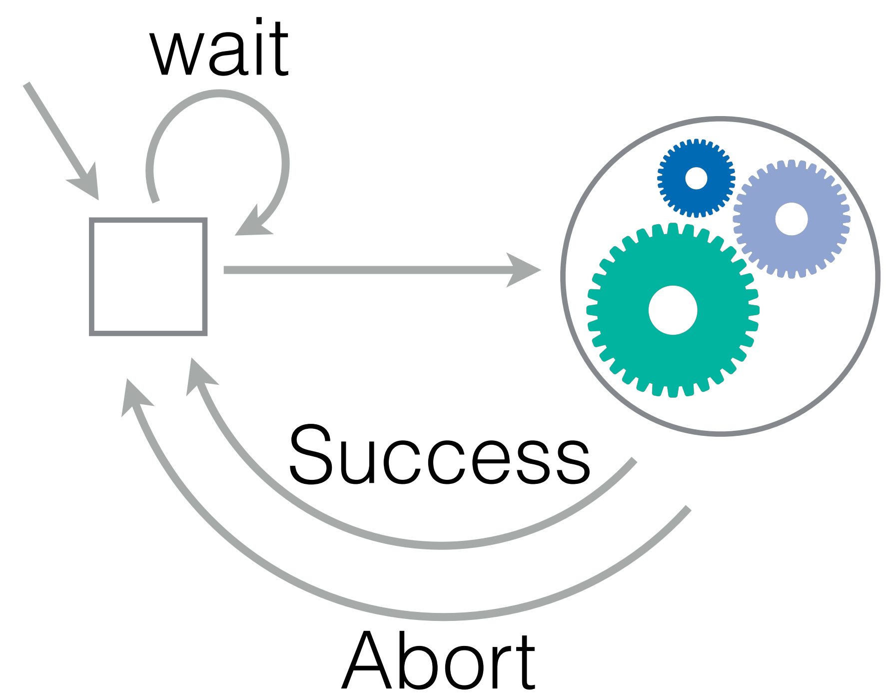
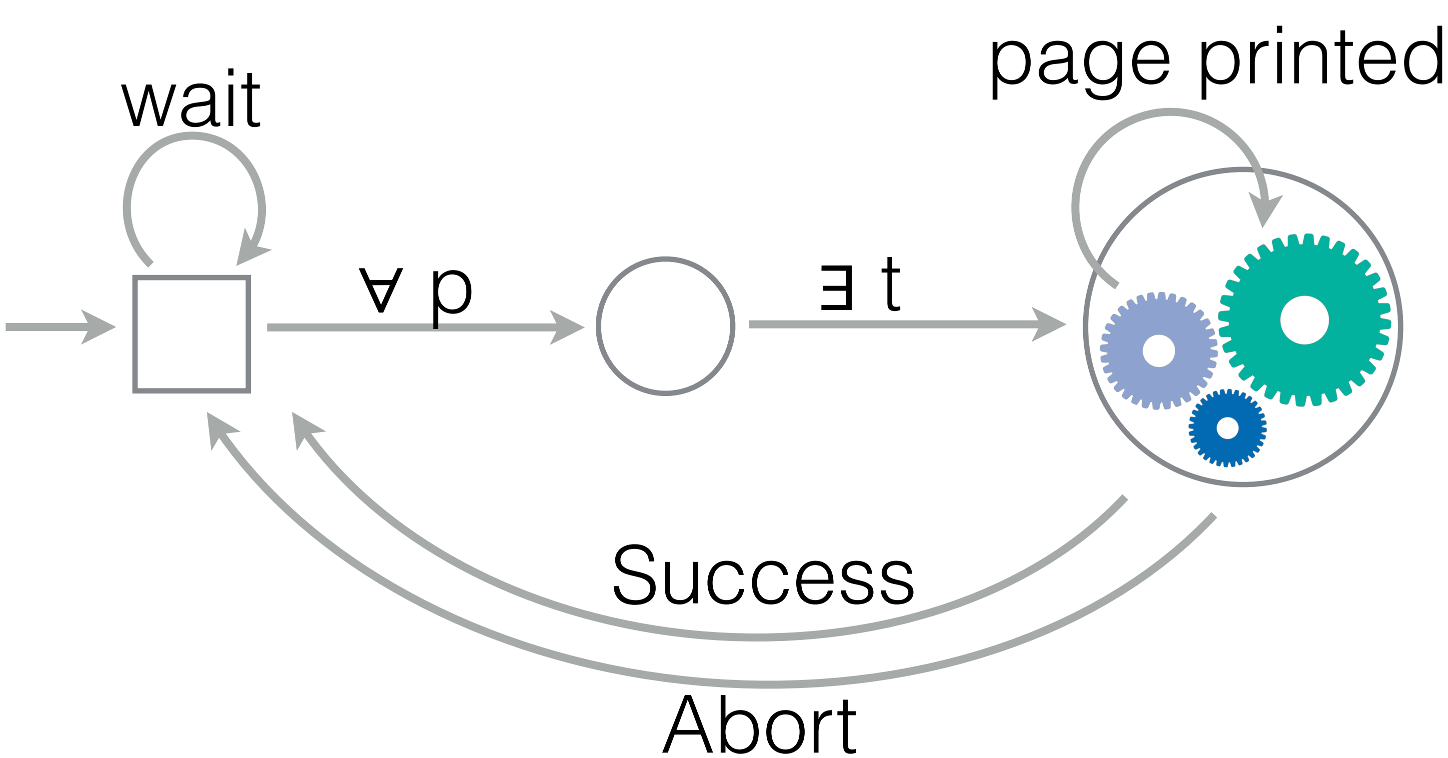
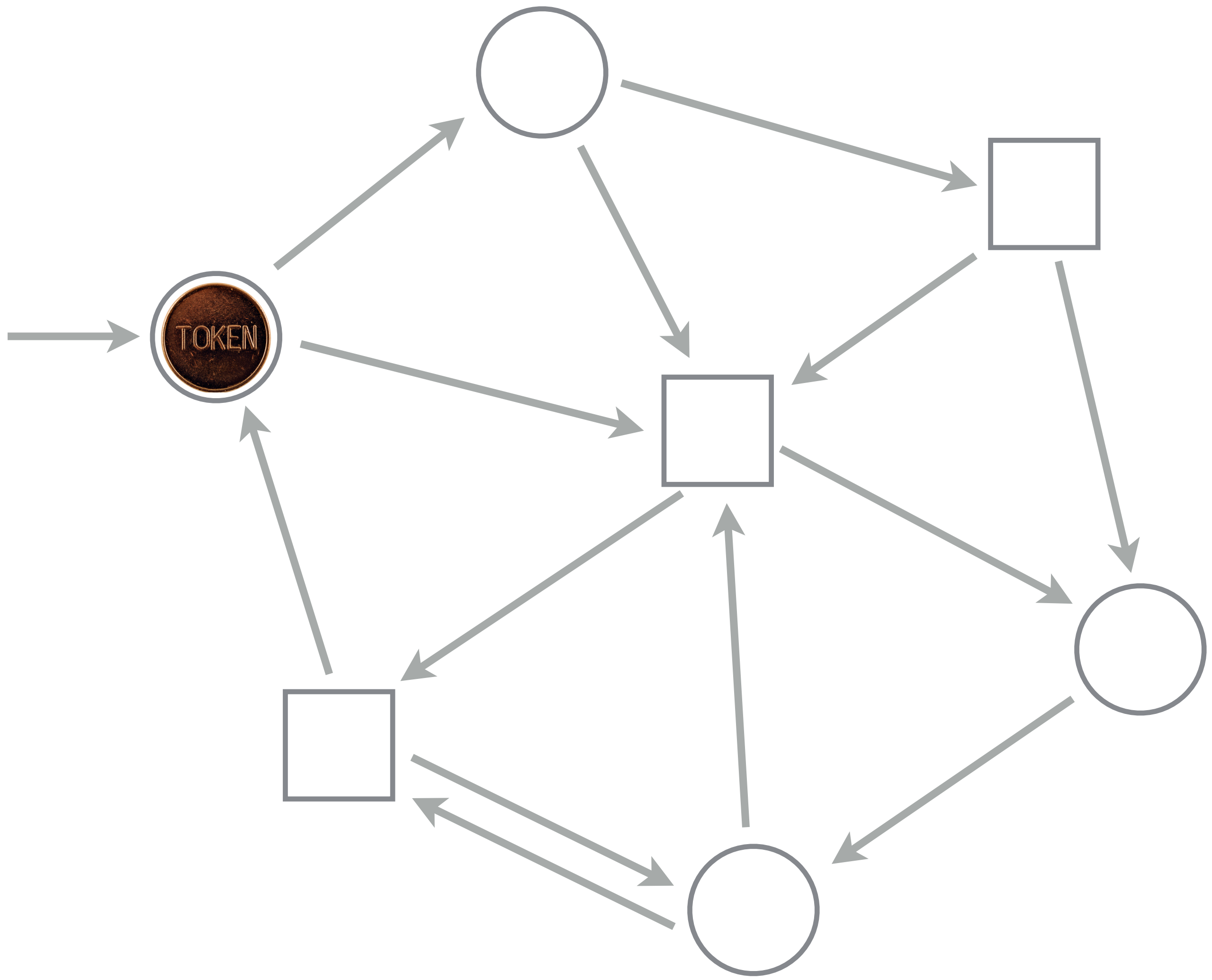




Games with bound guess actions can model things like:
- the user declares the number p of pages to be printed,
- the printer has to guarantee to bound the printing time by t, as a function of p.

Games with bound guess actions



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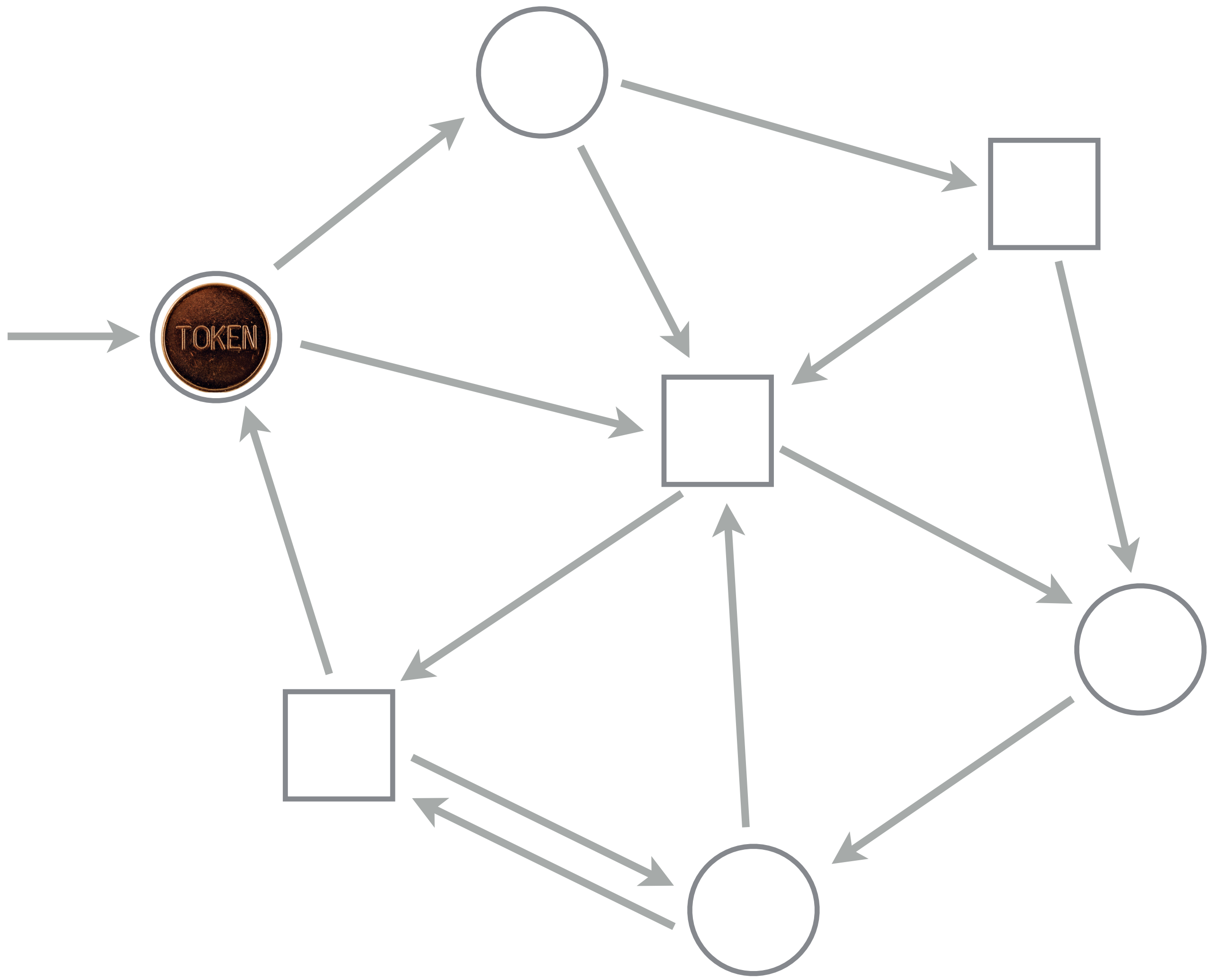

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A finite set of **registers** (r,s,t) is fixed (and are *owned* by the players ∃,∀).

Moves are labelled with normal actions or **bound guess actions ∃r , ∀s** (properly quantified).

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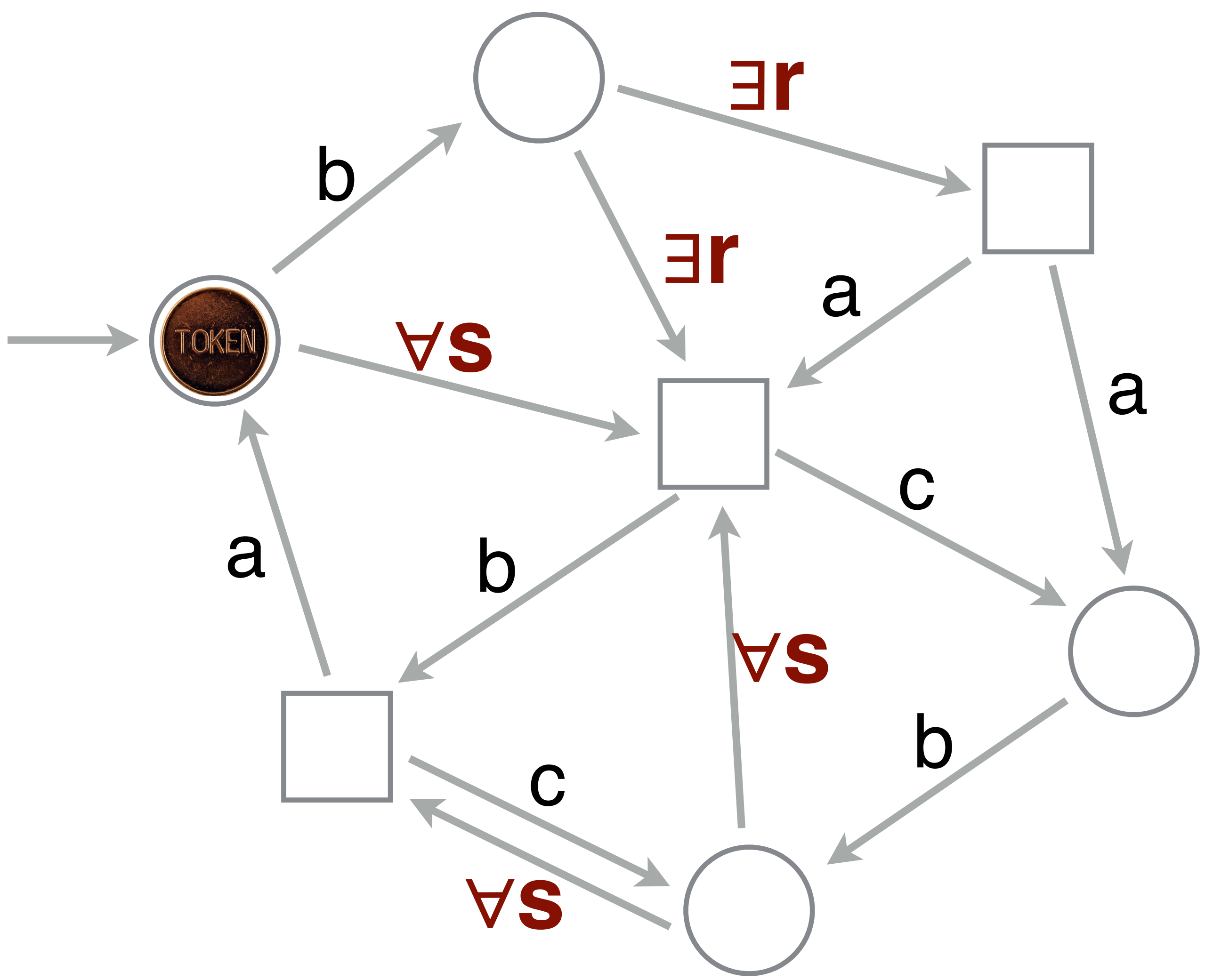

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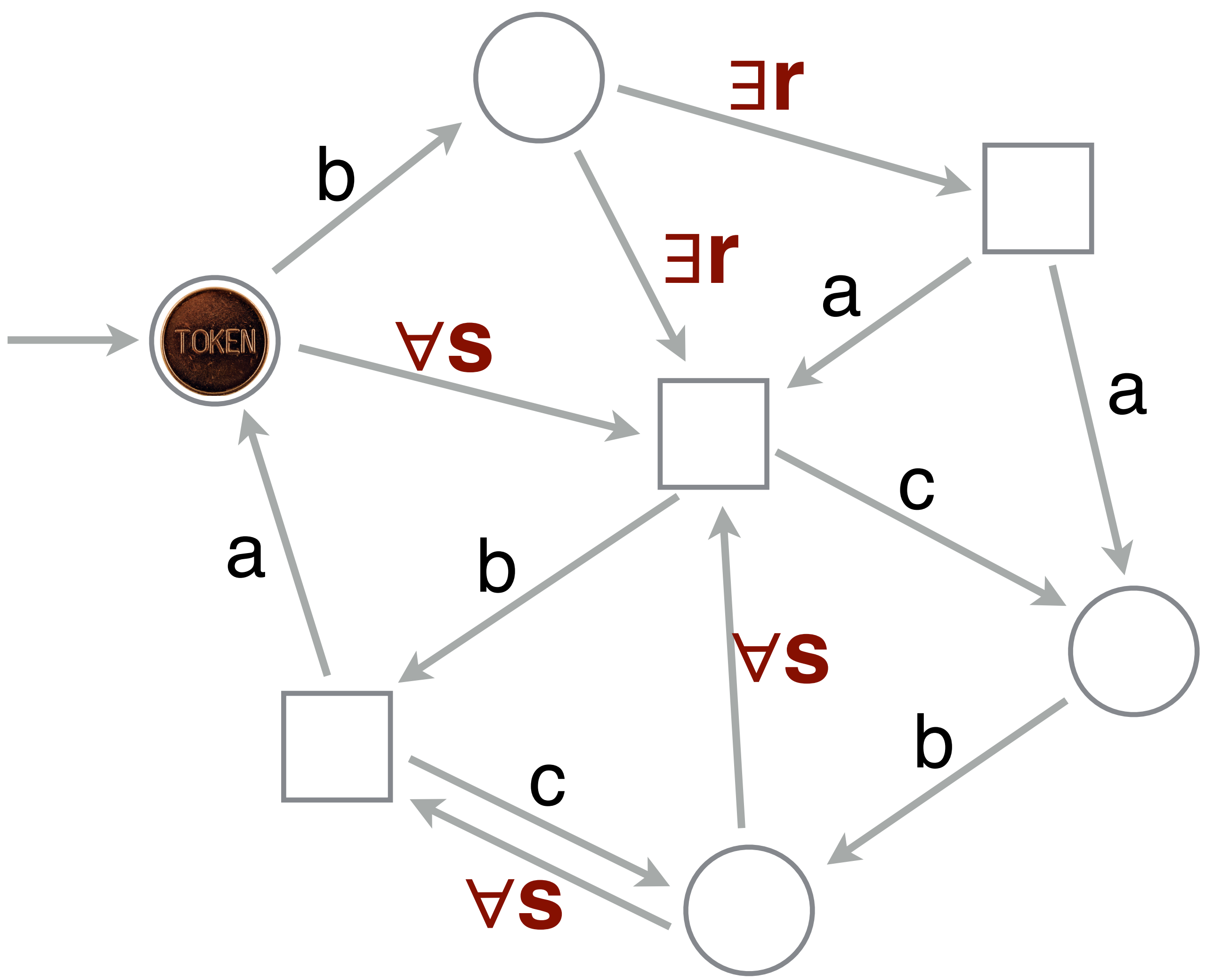

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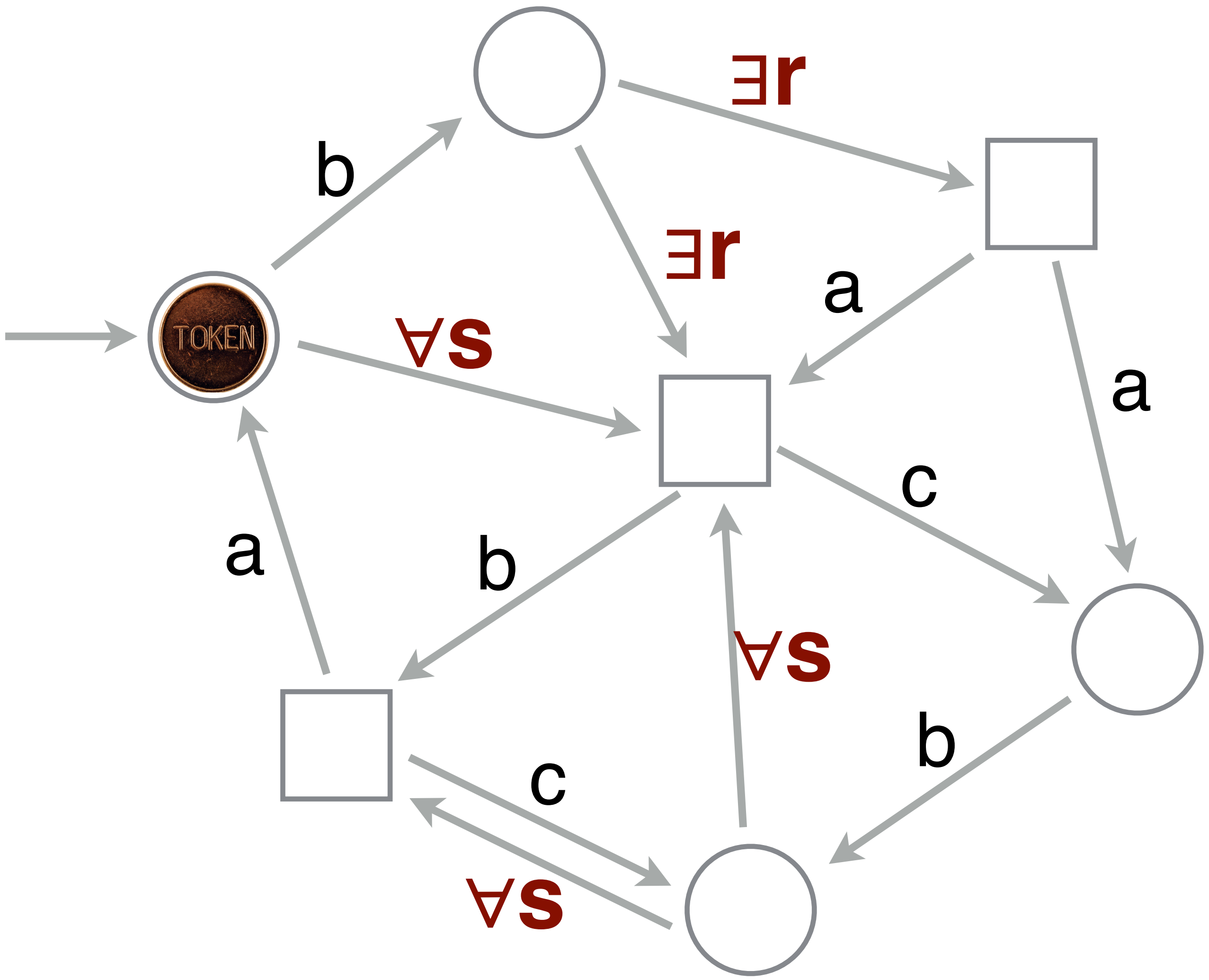

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- how some quantities exceed the current register values or not.

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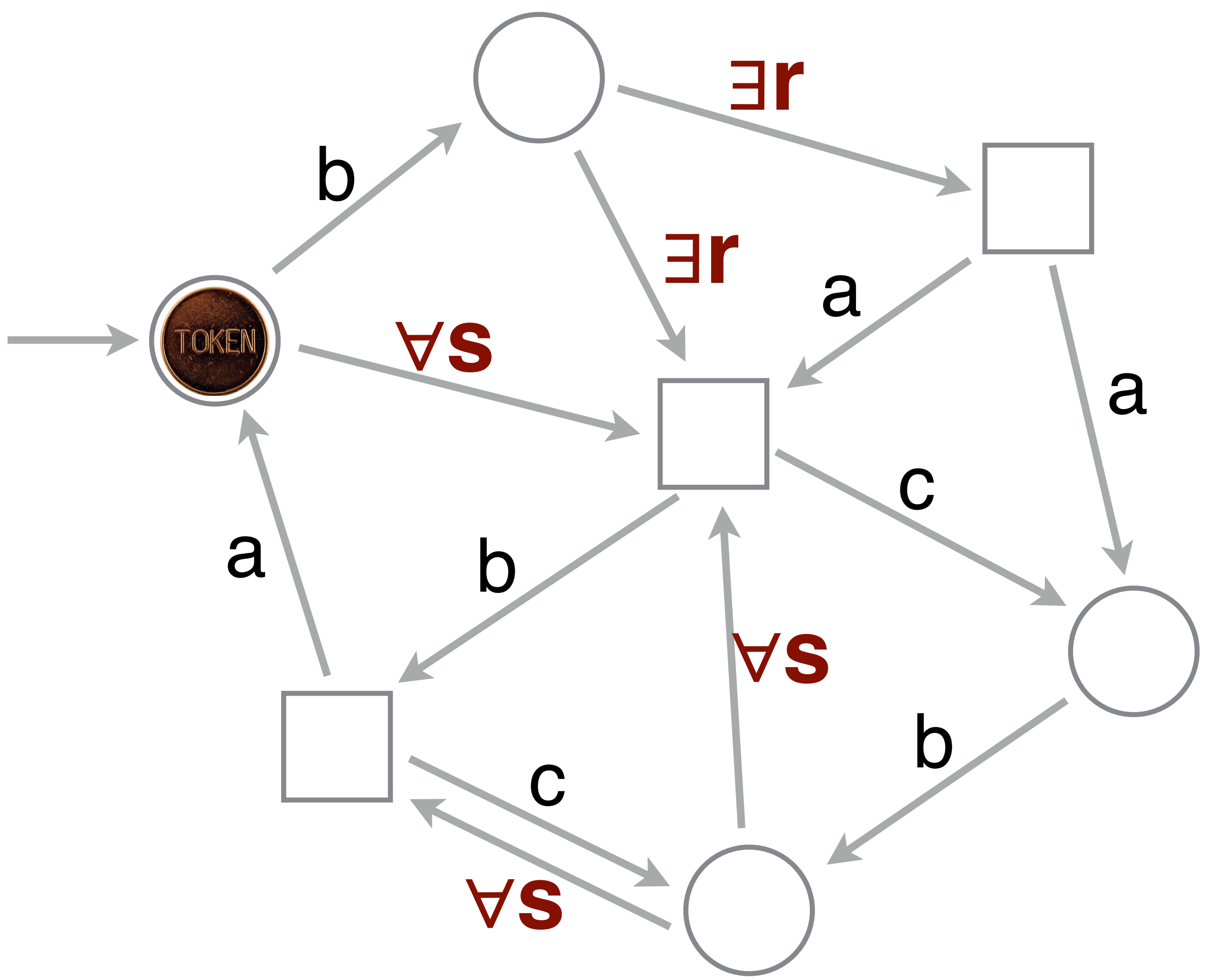

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Positivity: the chooser of the value aims at respecting the promised bound.

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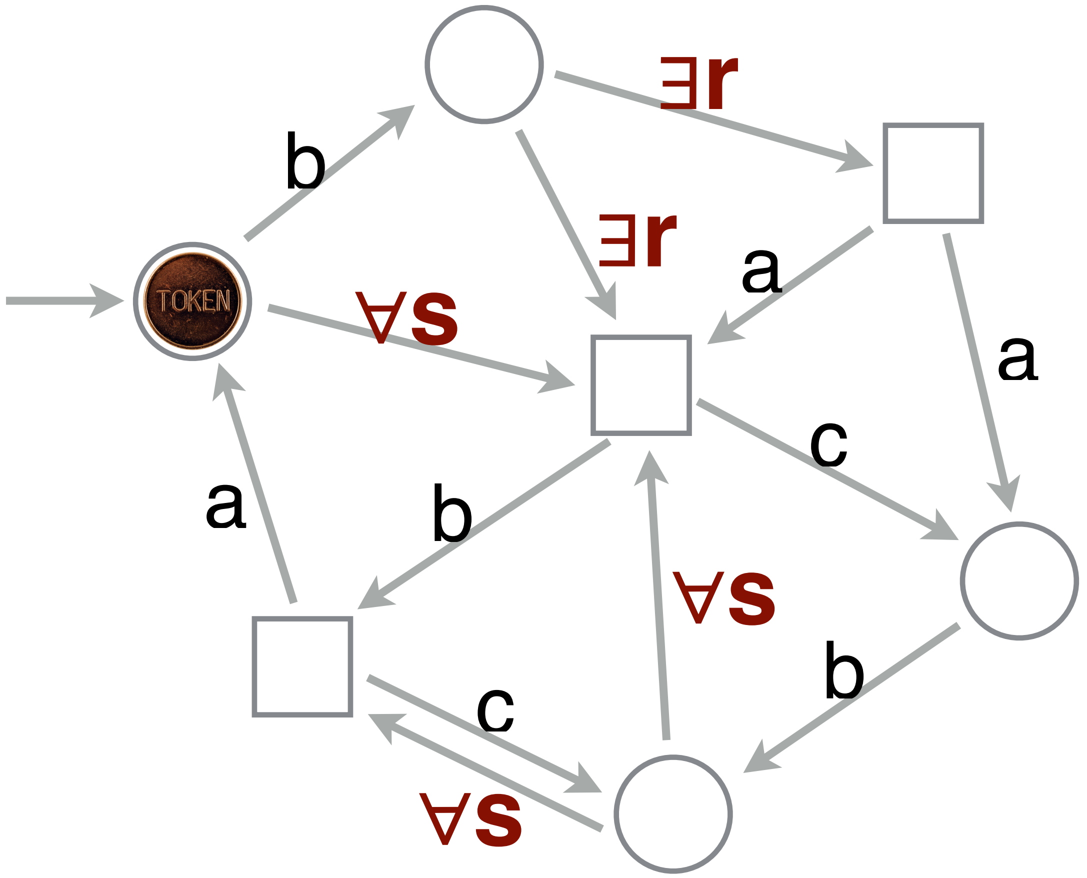

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- these are promises on the future of some quantity
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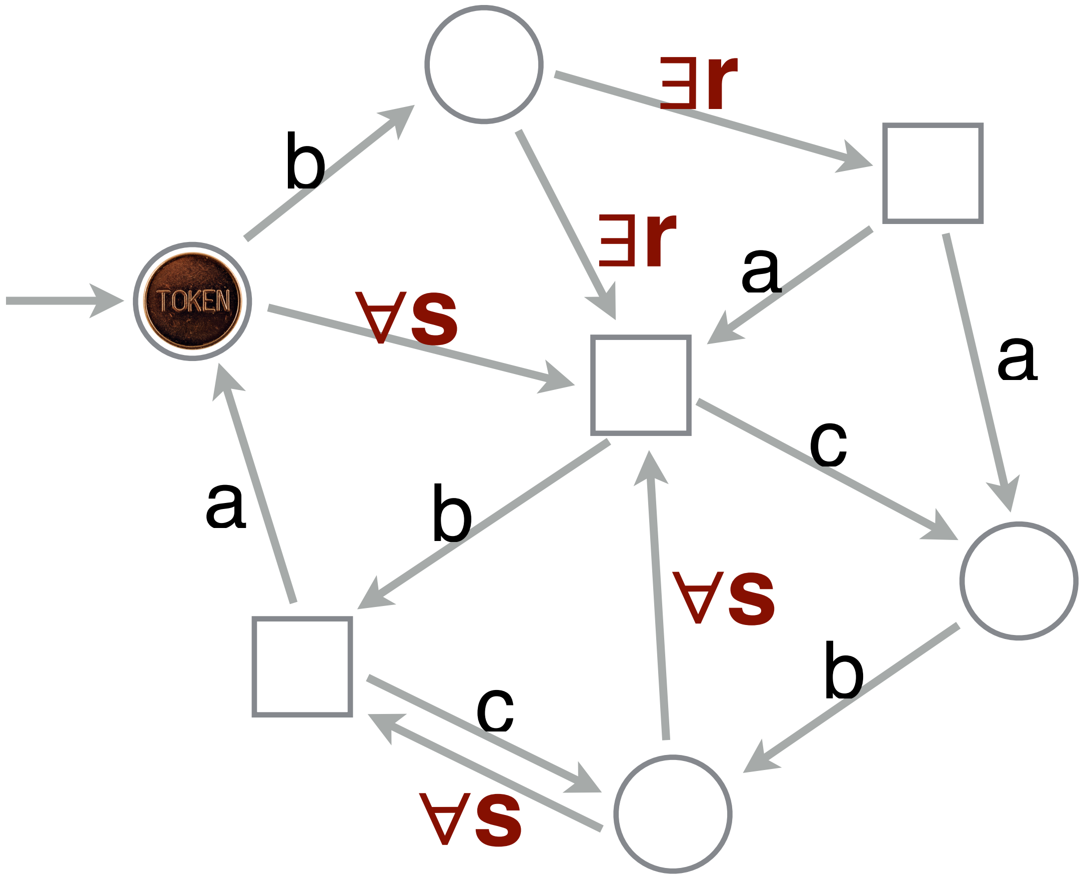

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What are the quantities ?

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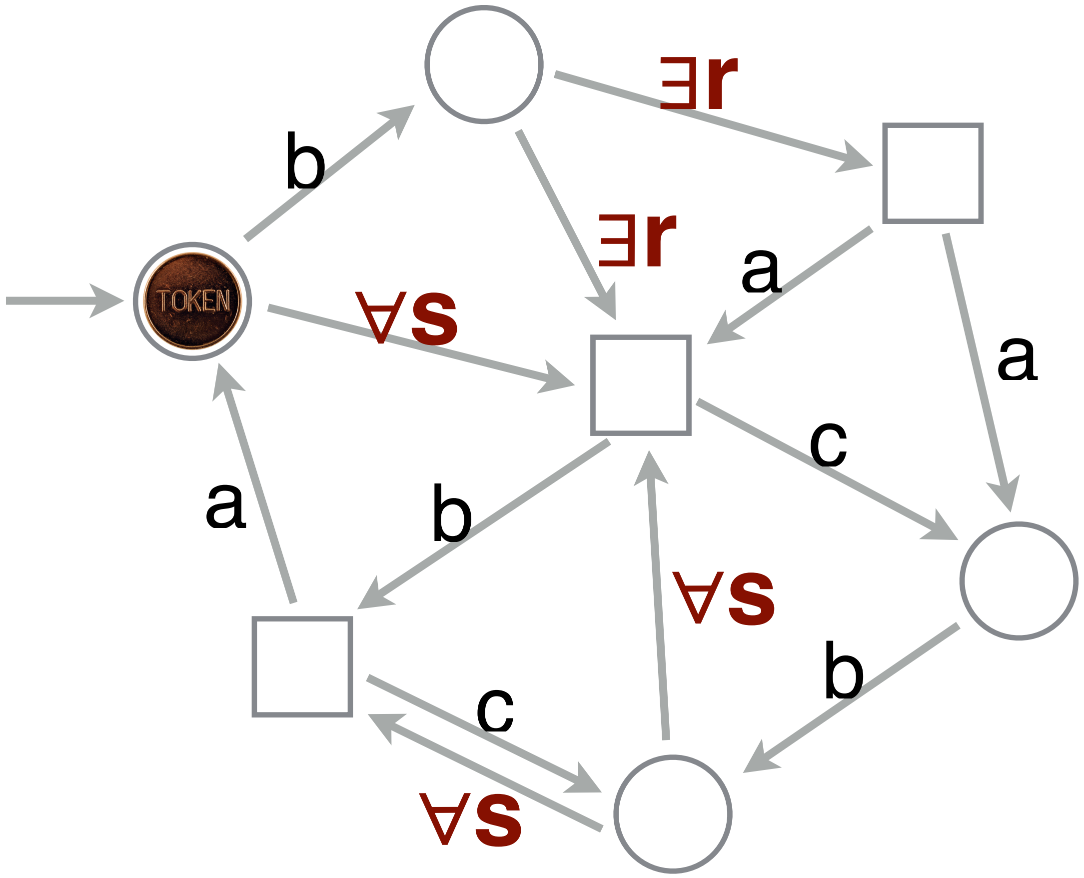

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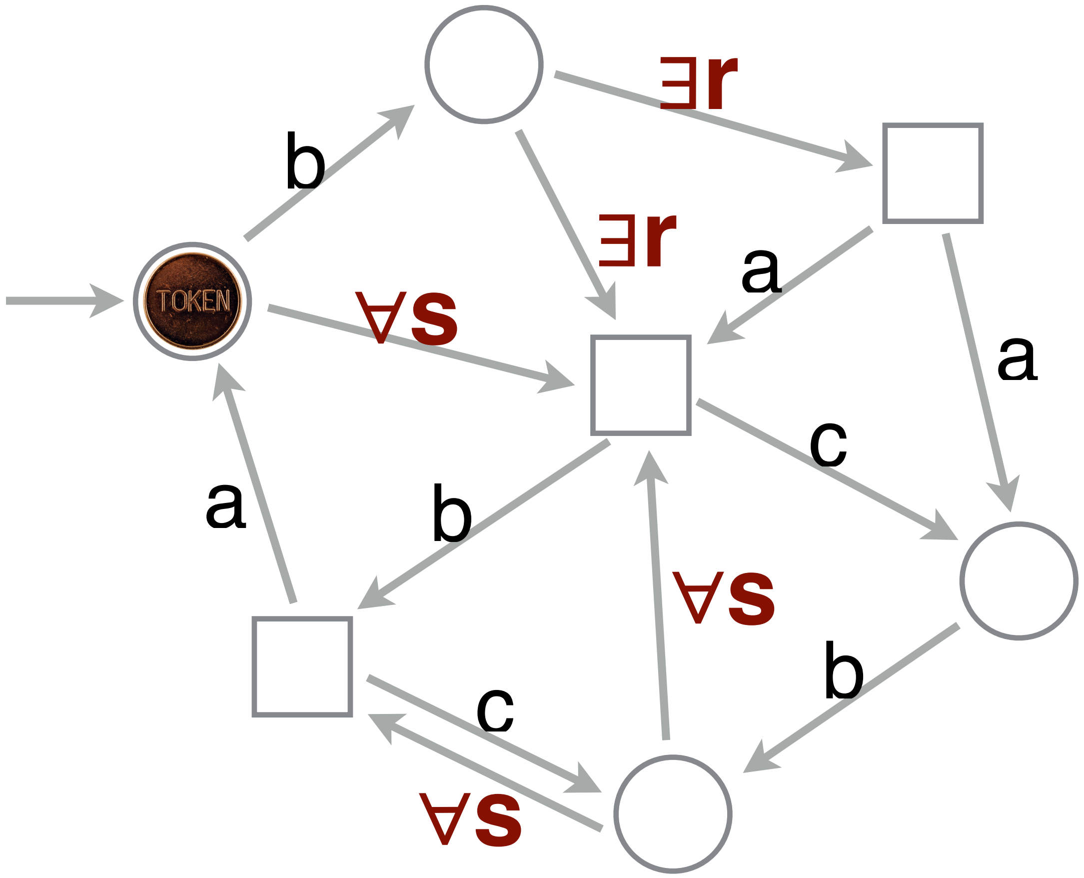

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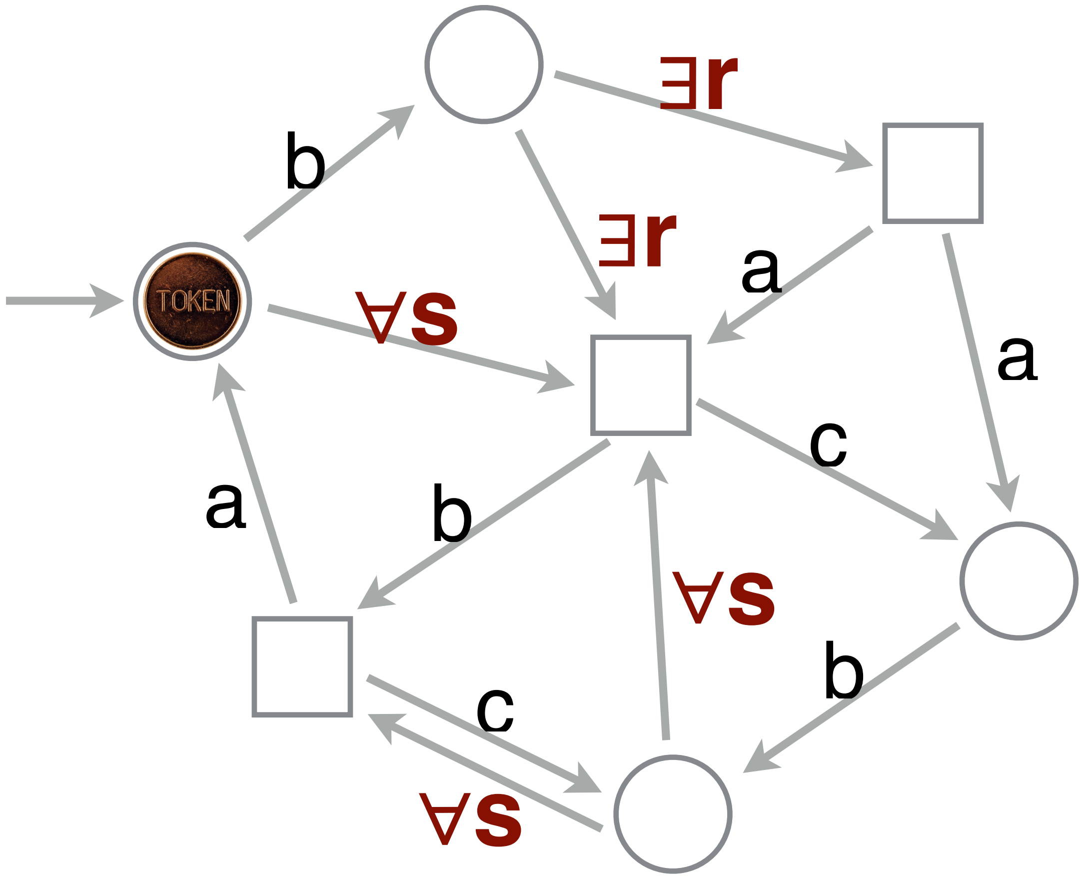

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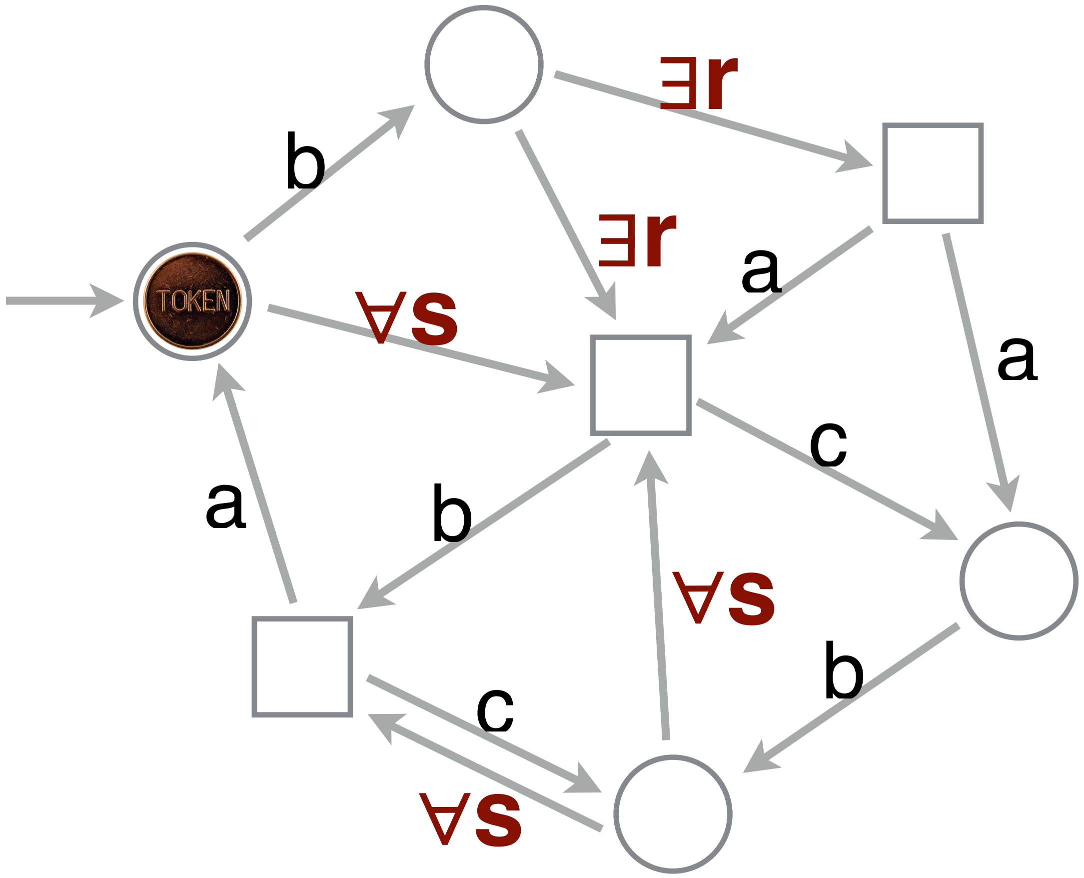

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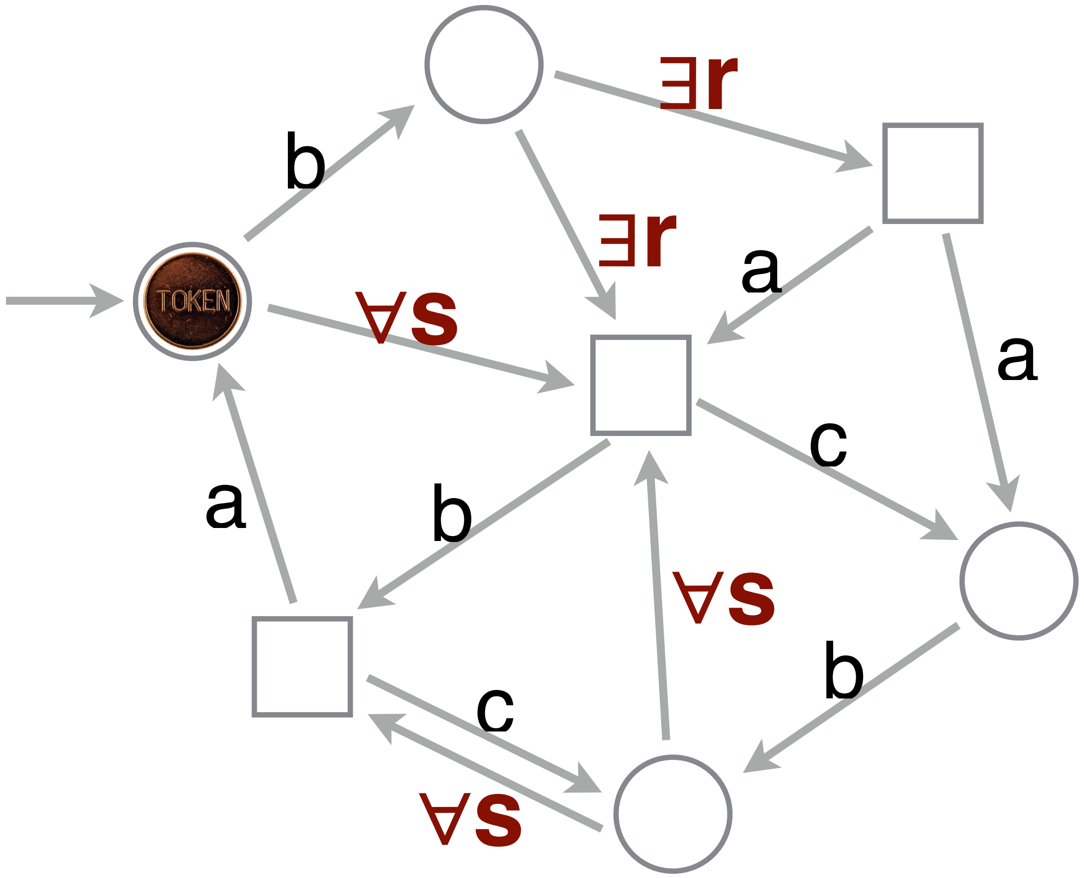

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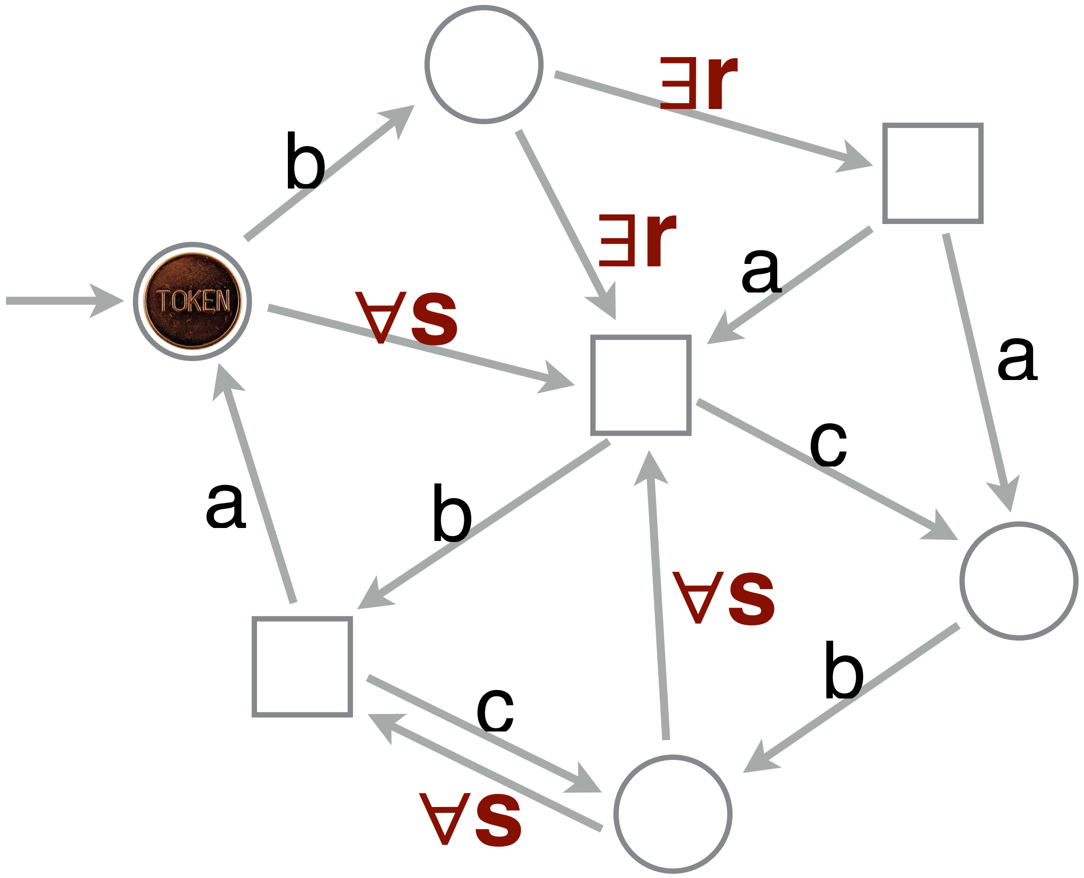

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« number of pages printed since the job was last initiated »
« largest number of consecutive action a seen so far »

The global condition is a regular language of words over actions enriched with bits representing « has quantity f exceeded register r ».
This bits have to be used positively.

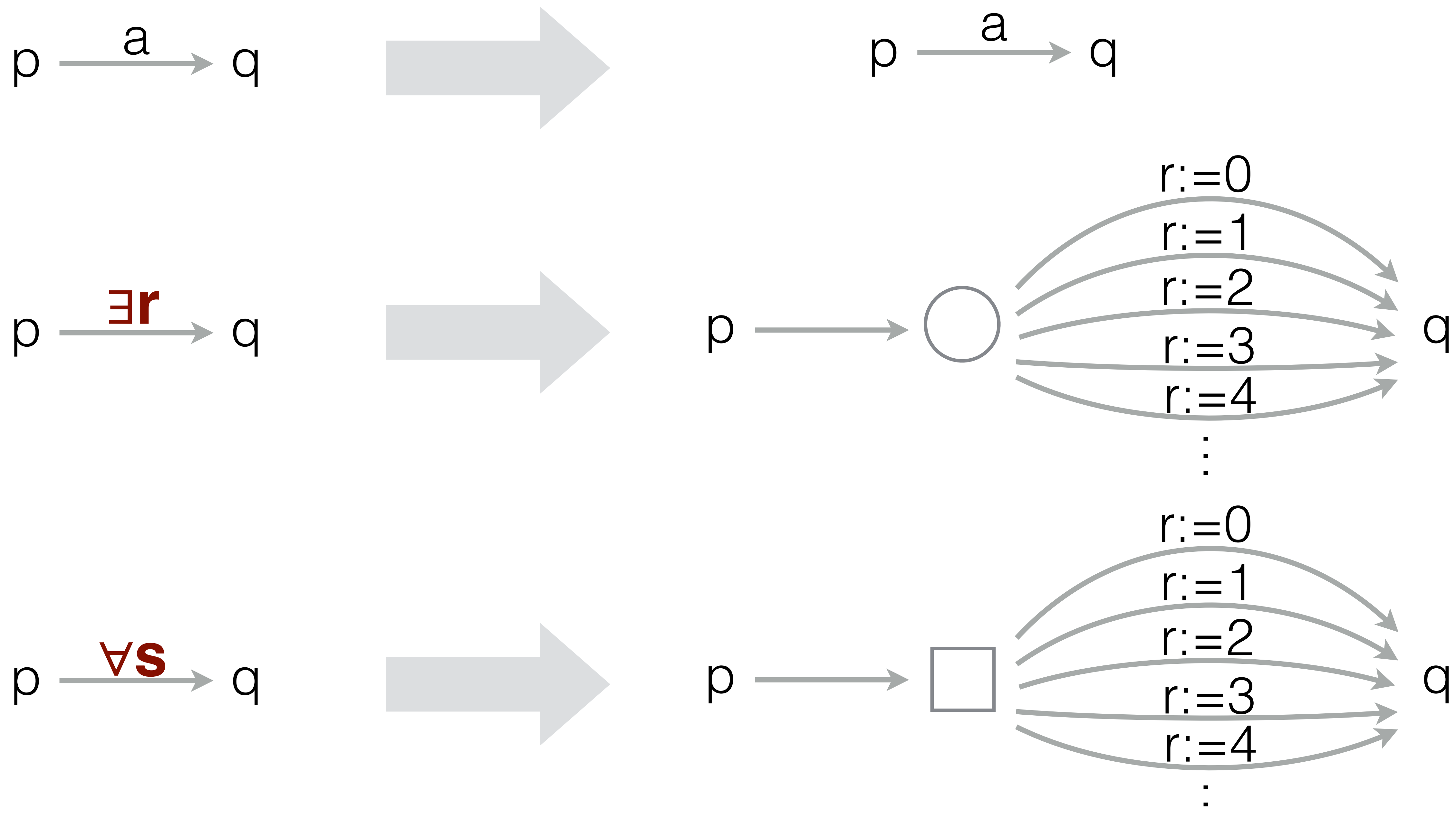
The result

Games with bound guess actions in general form:

- quantities = regular cost function
- global condition = any ω-regular language (positive)

Theorem: The winner of a finite game with bound guess action in general form can be decided.

Translation into usual games



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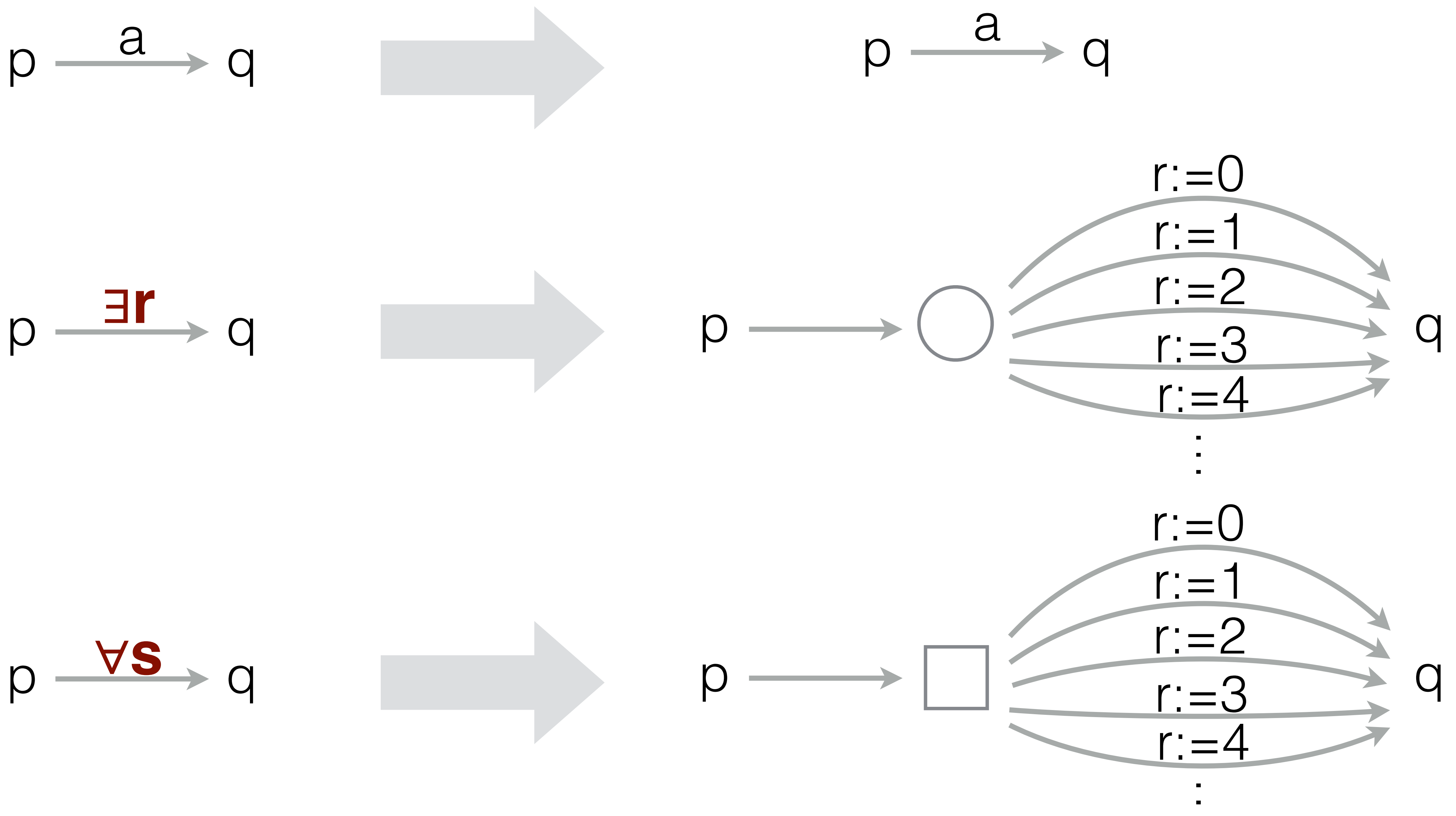

Formally, this translation is a way to describe the semantics of games with bound guess actions.

Strategies in games (wbga)

Strategies are used to define the property of being winning.

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(Standard) strategies (for the existential player) are trees with vertex labelled nodes, such that

- either a vertex is owned by the existential player, and it has one child…
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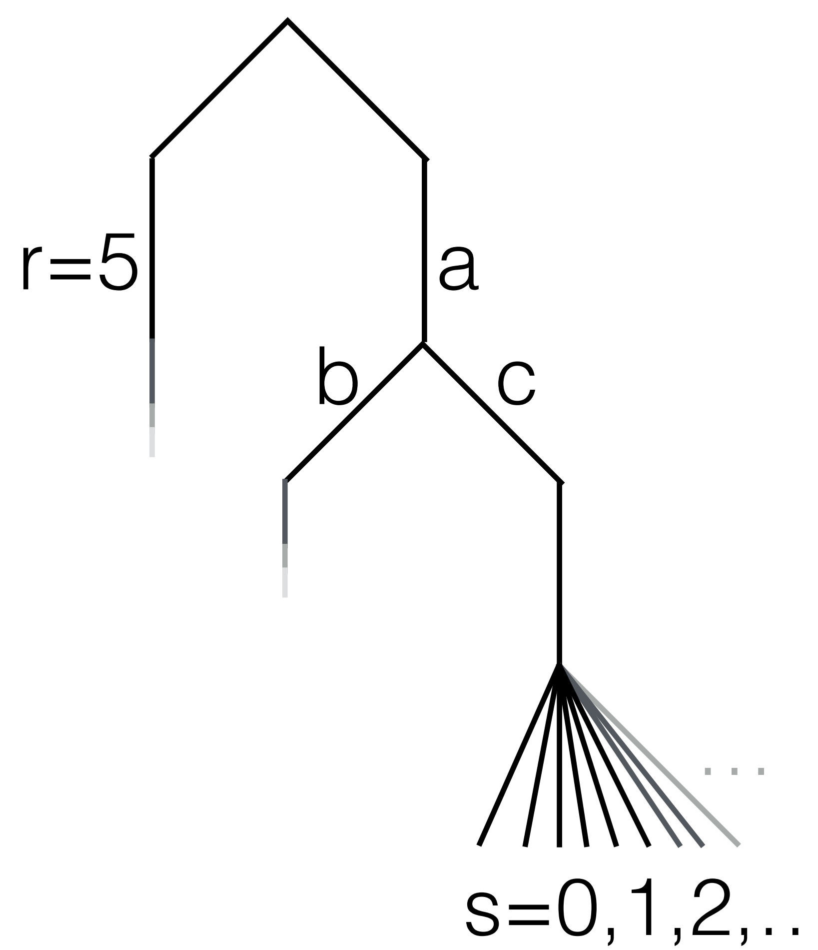

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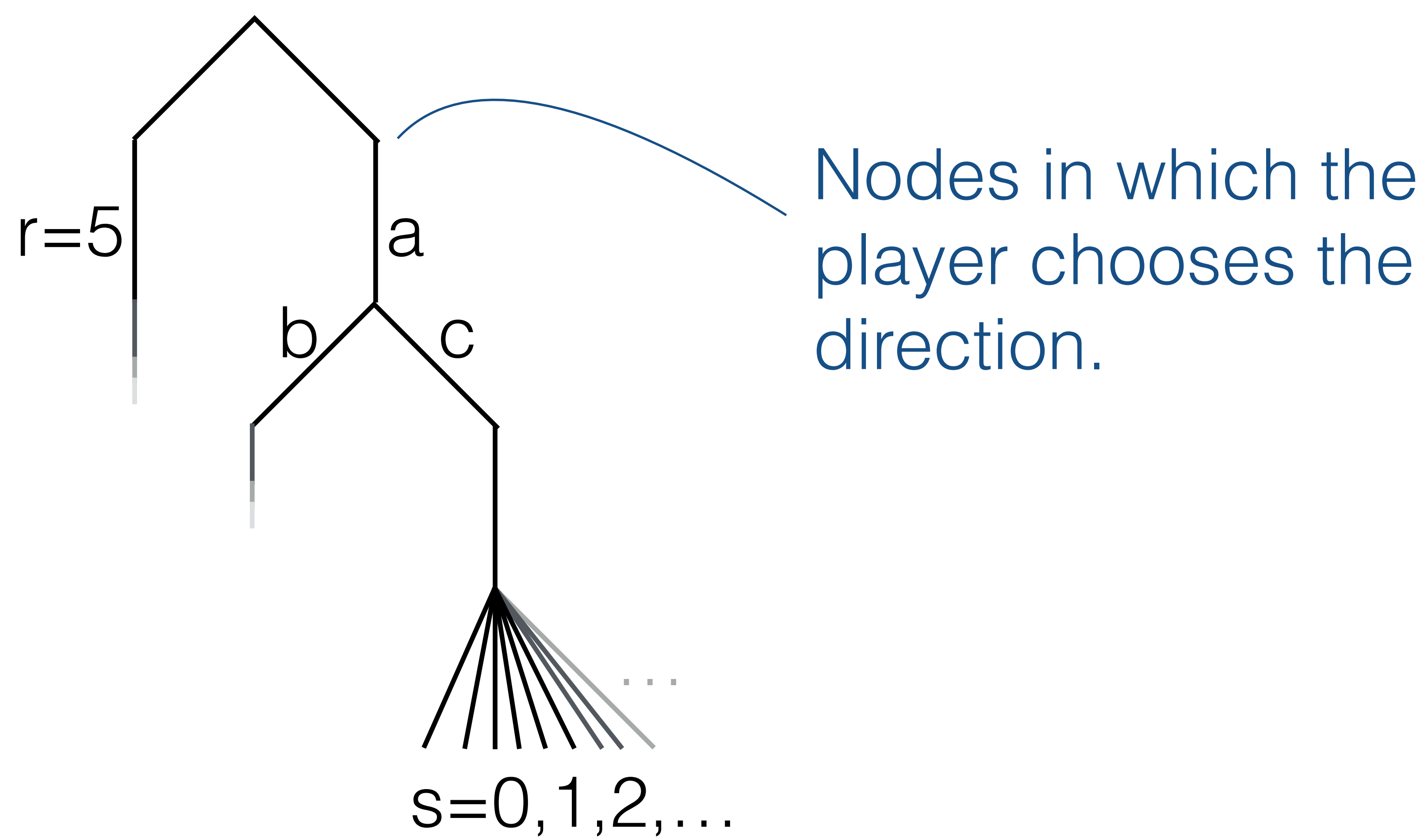

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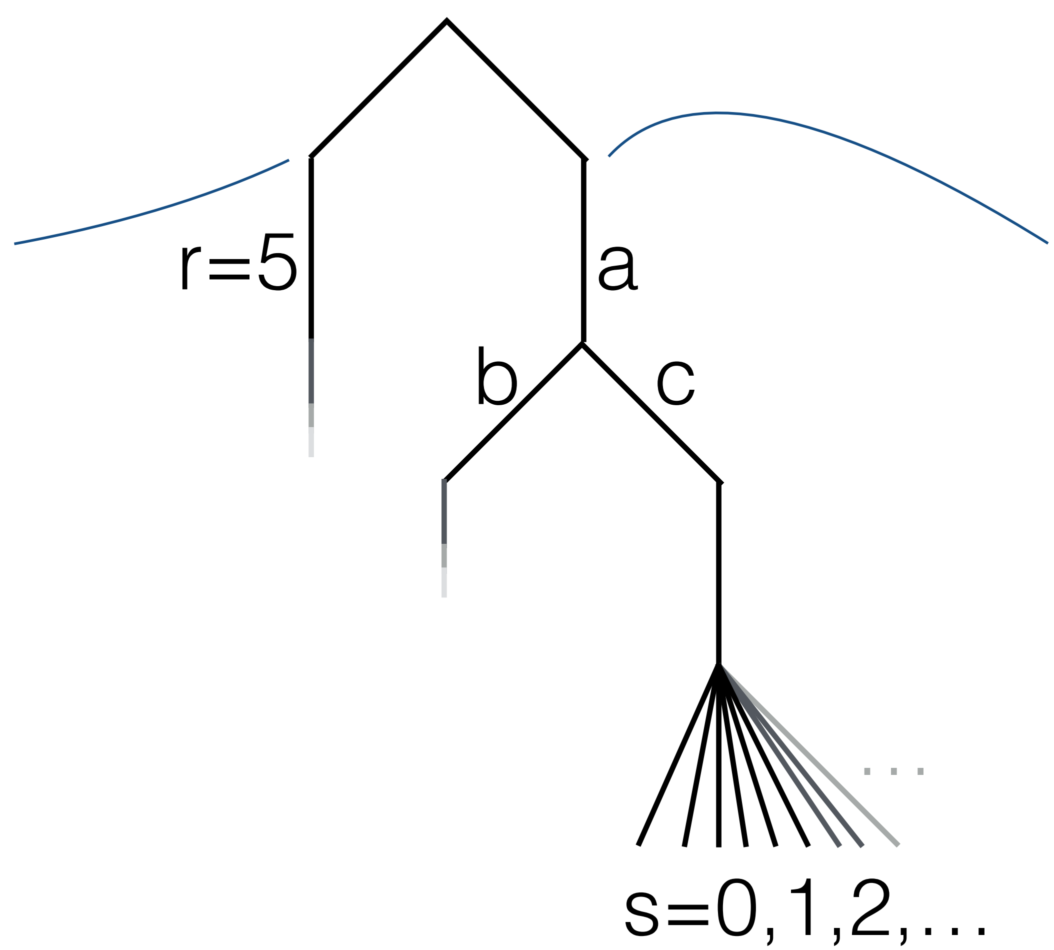
Nodes in which the player chooses a value of a register



Nodes in which the player chooses the direction.

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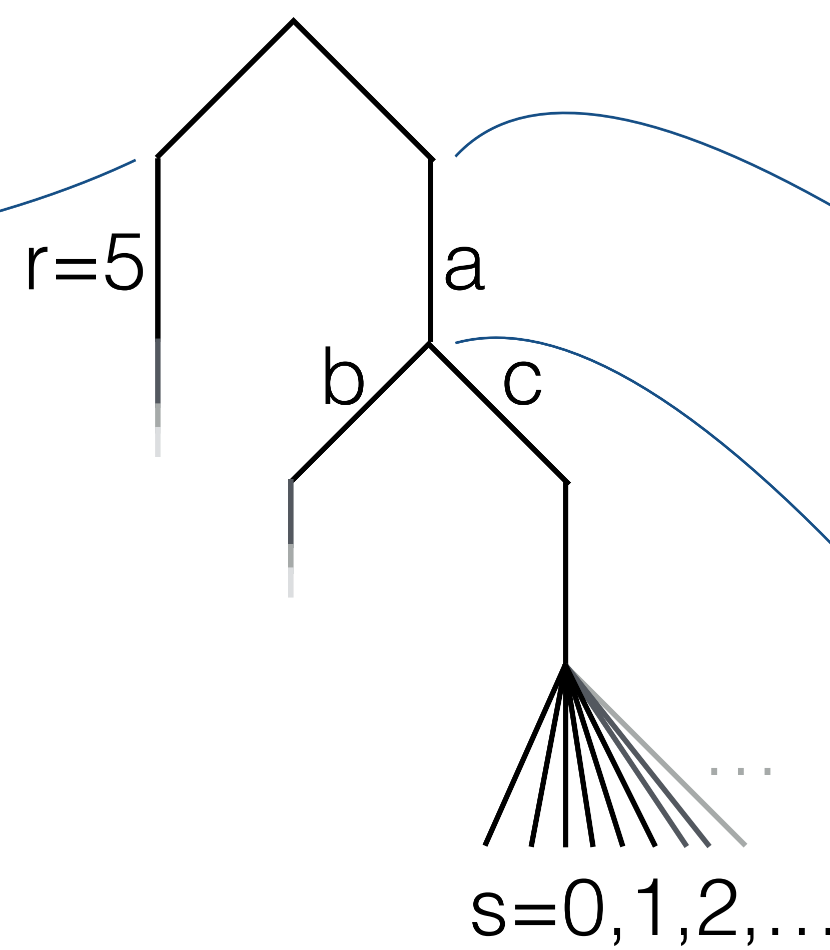

Nodes in which the player chooses the direction.

Nodes corresponding to the opponent choosing the direction.

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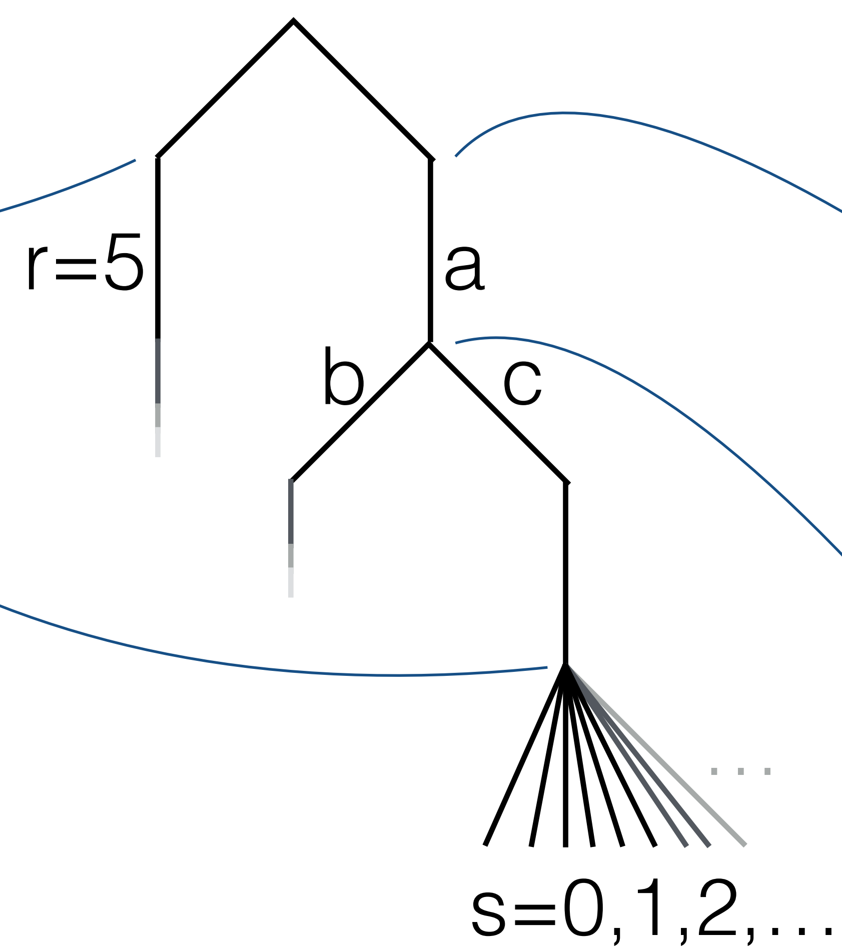
Nodes in which the player chooses a value of a register

Infinitely branching nodes in which the opponent may choose any value for one of its registers.



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Simple games with bound guess actions:

- quantities = max over several counters ɣ of

  « the number of **inc**$_\gamma$ since the last **reset**$_\gamma$ or the beginning of the word »
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Lemma(reduction 2): A finite simple game with bound guess actions can be effectively turned into a finite ω-regular game with of same winner.

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Main change

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- being definable in cost monadic second-order logic (costMSO)
- being described by a B-automaton, an S-automaton,
- being described a B-regular expression, or an S-regular expression,
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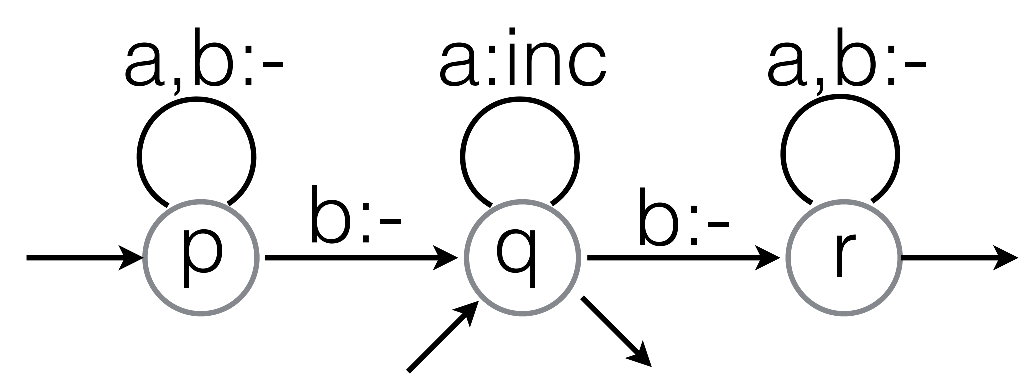
A B-automaton has counters that can be incremented or reset
It accepts a word with value n if there exists an accepting run such that no counter exceeds value n.



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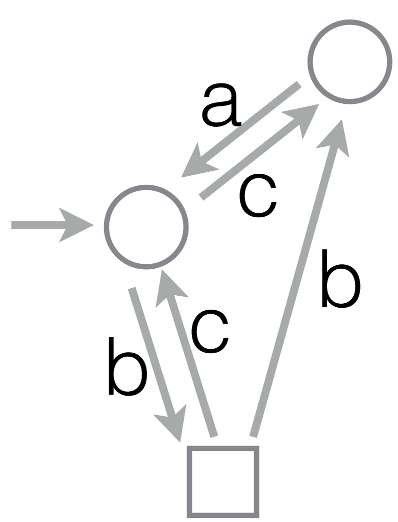

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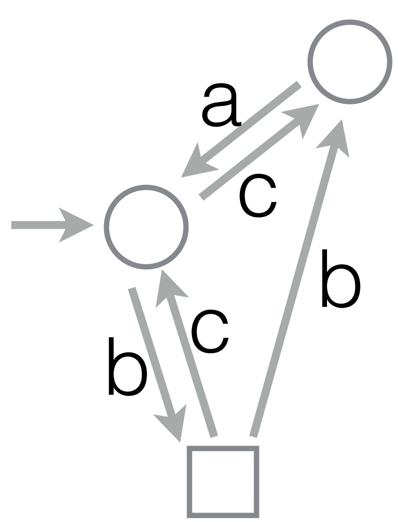
- W-automaton for L: W is the accepting condition; L is the accepted language
- W-game: W is the winning condition



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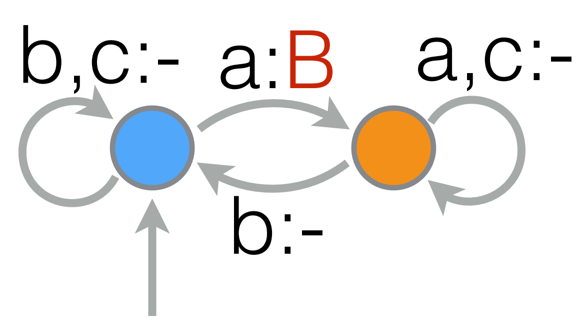

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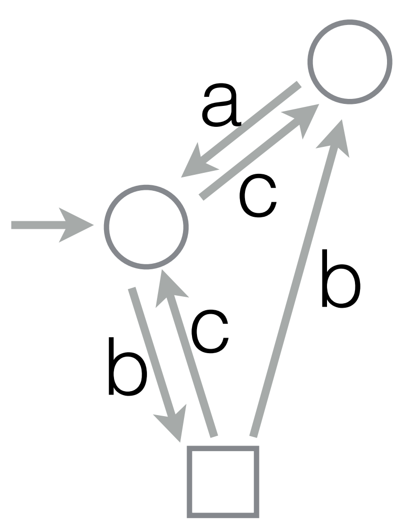

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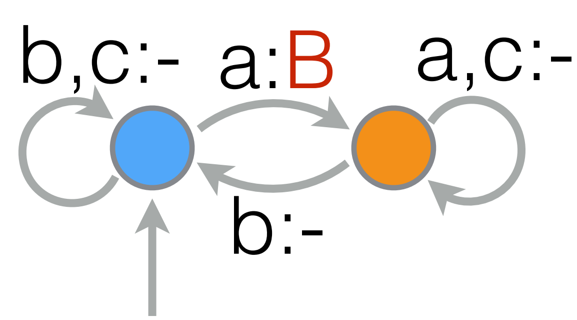

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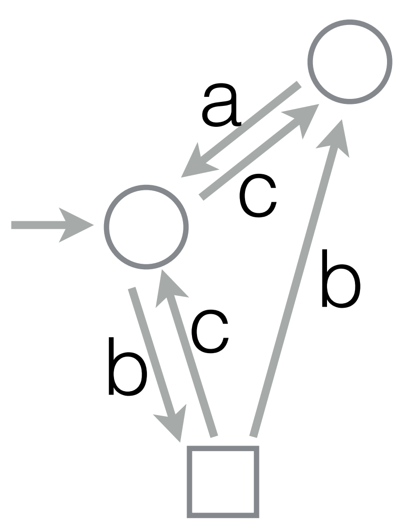
the winning condition (L-game) — the accepting condition / the accepted language (W-automaton for L) — the winning condition (W-game)



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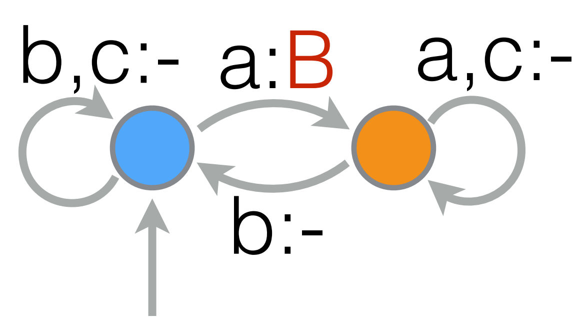

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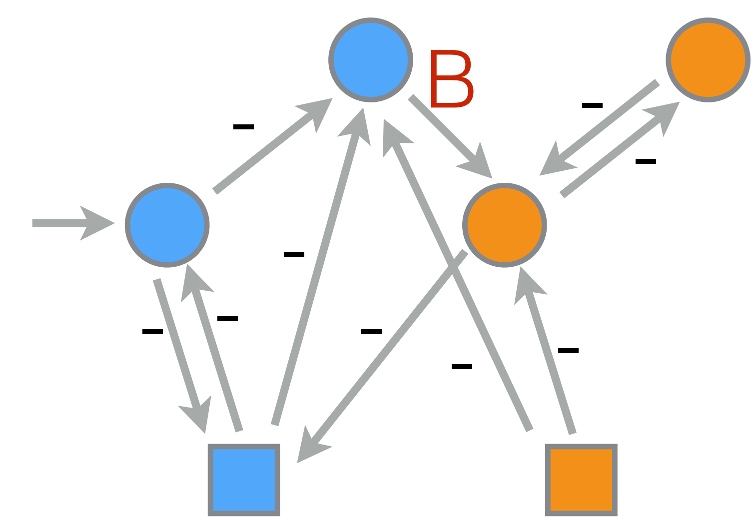

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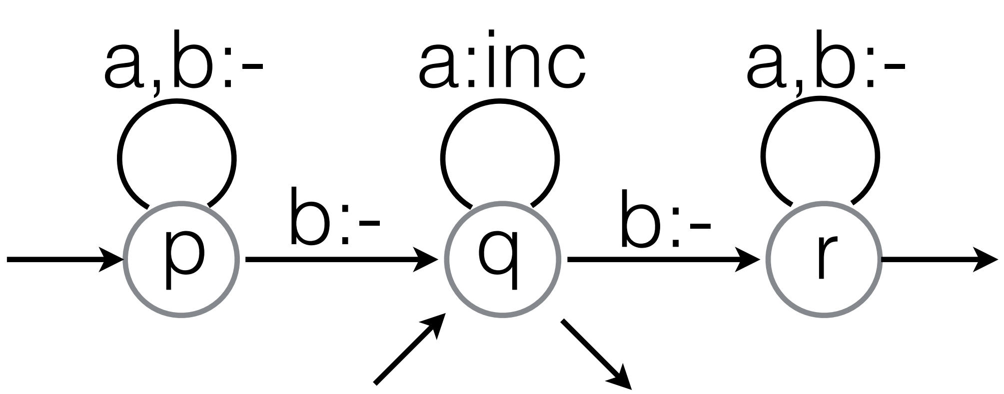

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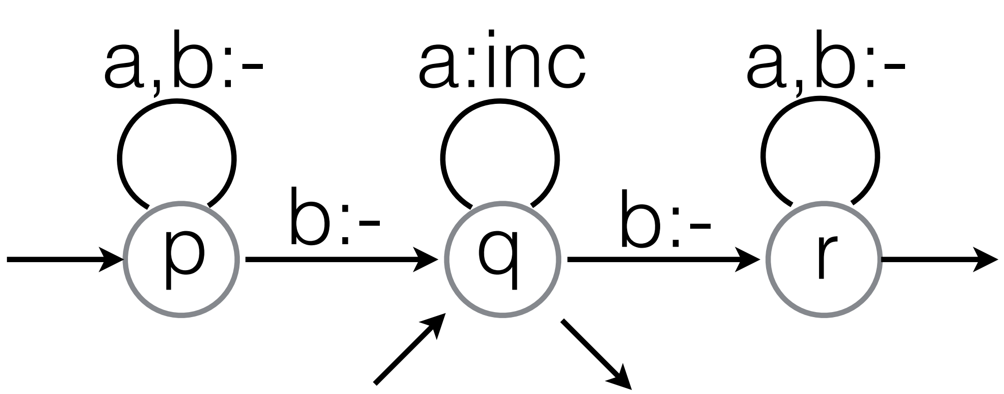

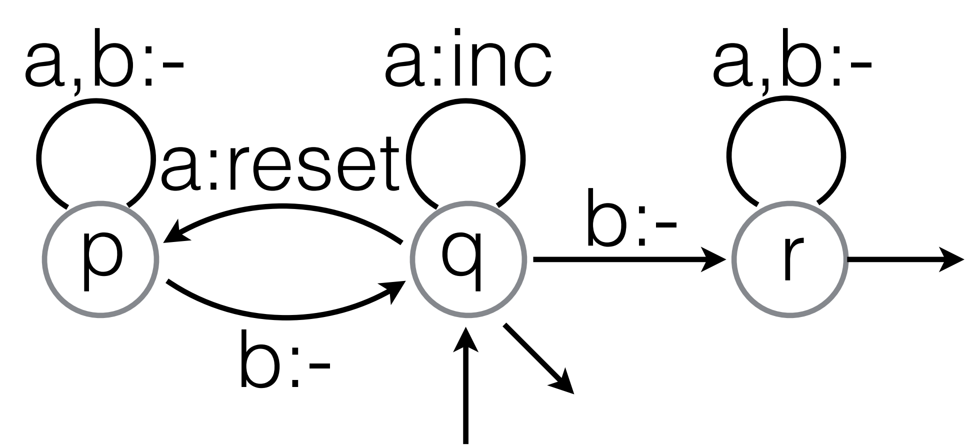

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by maintaining a permutation of the registers one may « know » during the game what is this order.

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Using the permutation or register techniques, one can « essentially » restricts to a situation where
1) the registers are not guessed anymore,
2) their relative order (of magnitudes) is known.

From simple to ω-regular

We assume r1 « r2« r2« … « rk known (as if bound guess actions at init).

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Corresponding ω-regular condition:

- if there are infinitely many **inc1**, finitely many **reset1**, then owner1 looses, else
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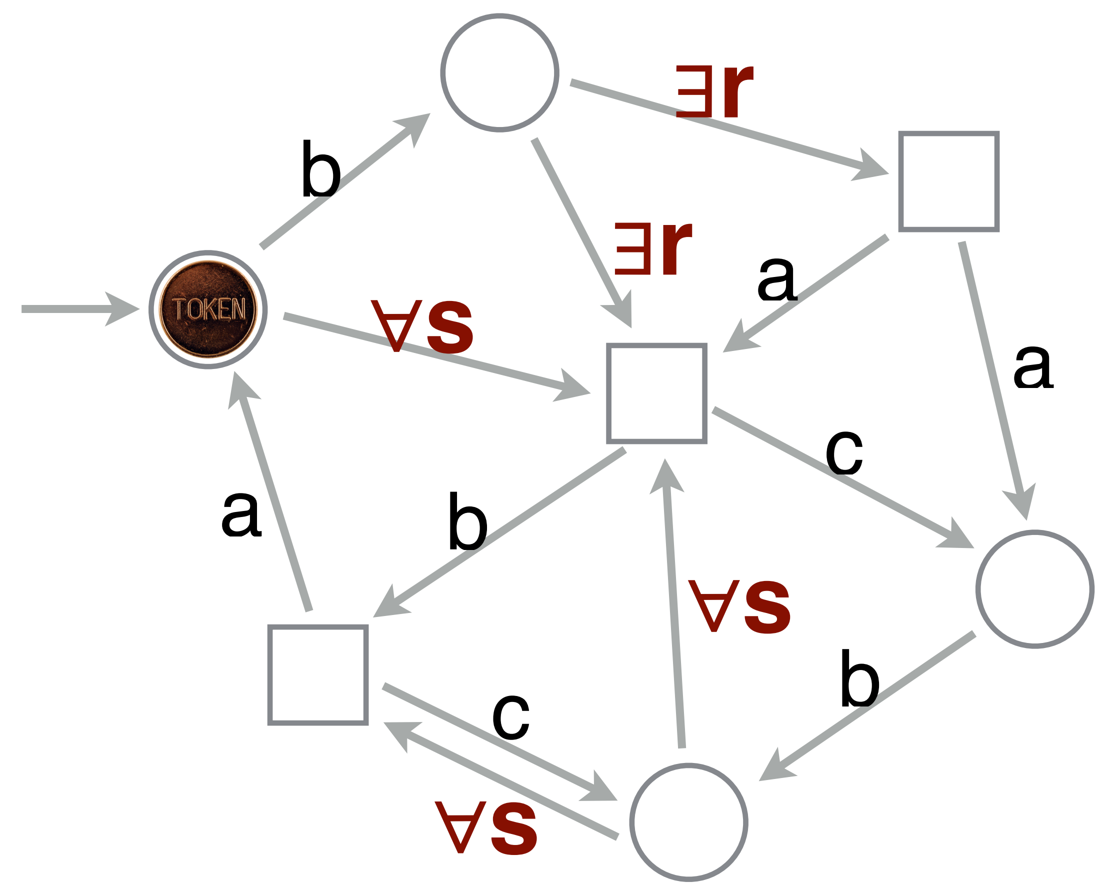
The proof crucially uses the finiteness of the game, and the existence of finite memory strategies in ω-regular games.

Conclusion

Games with bound guess actions allow to describe phenomenon that virtually happen in infinite games.



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Finite such games with a reasonable class of conditions
- regular cost functions as quantities,
- regular condition as long term goal,

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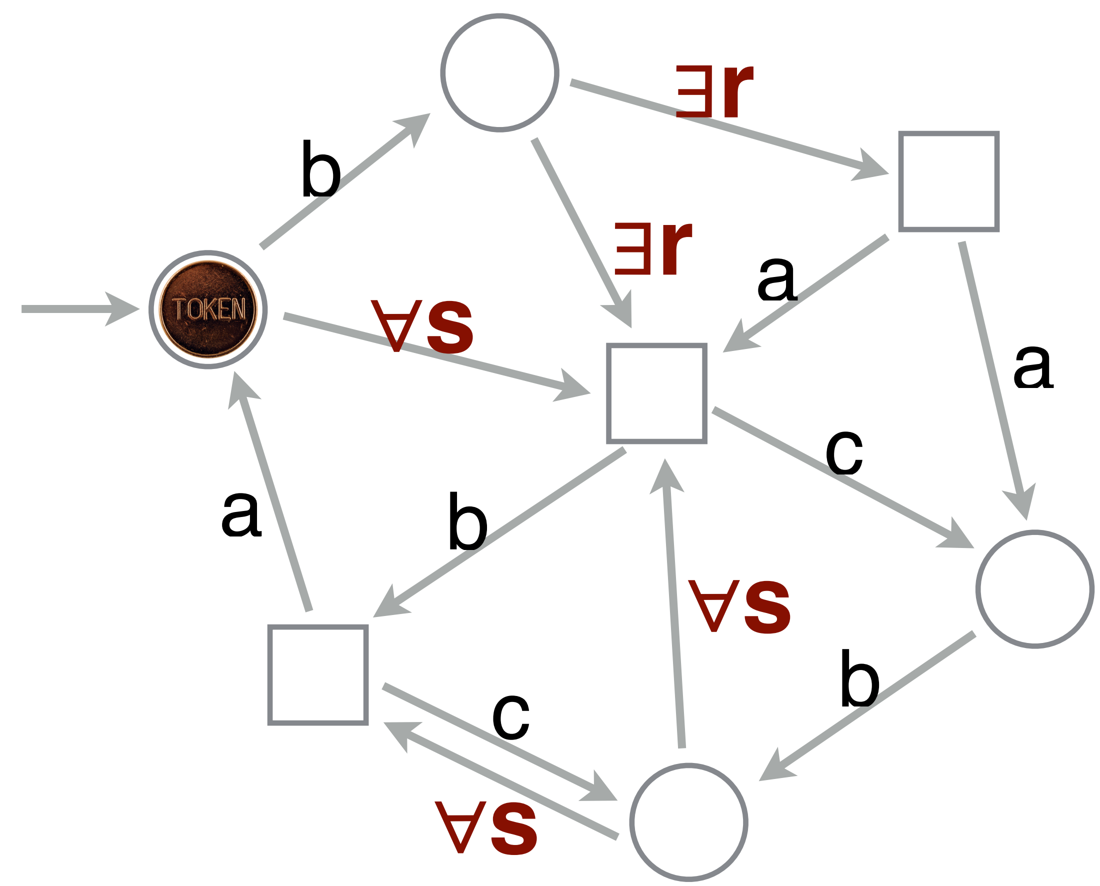

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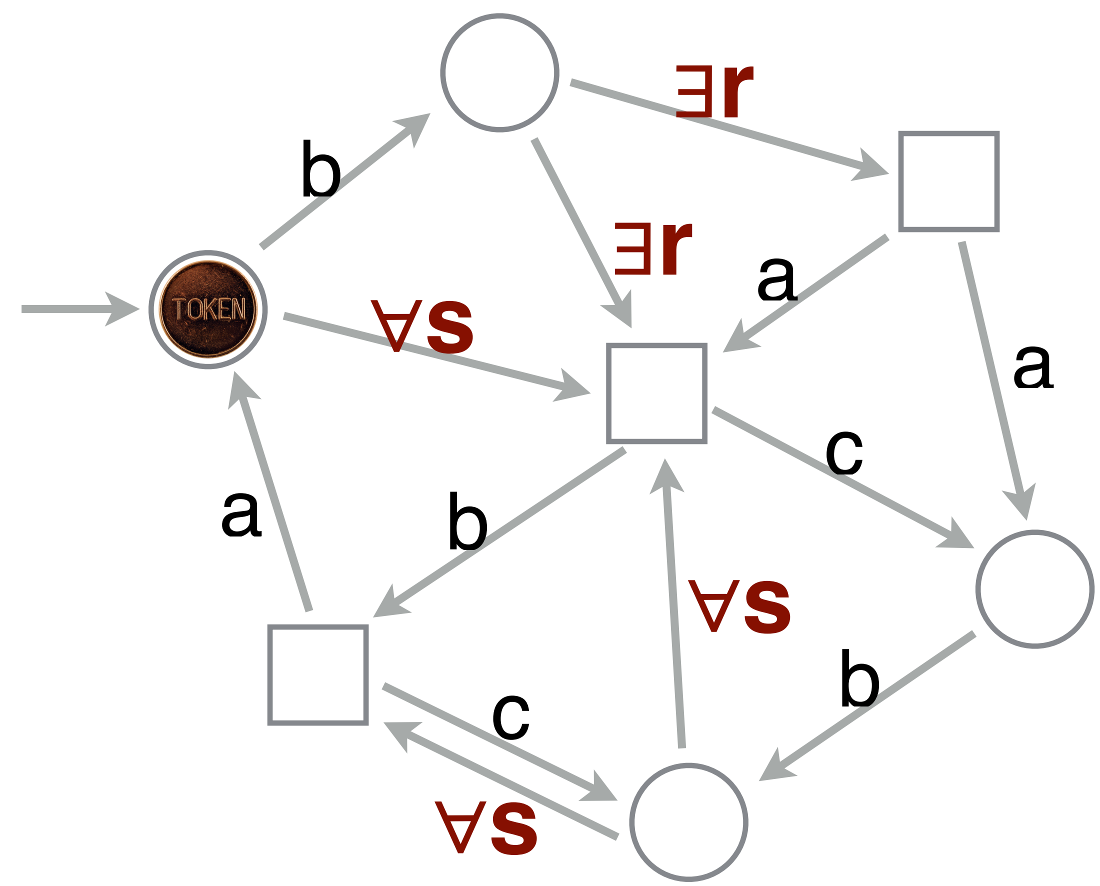

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The proof goes into several step of reduction involving:

- history-deterministic cost automata,
- LAR-like technique for assessing relative magnitudes of register values,
- a final reduction to ω-regular condition.