Combinatorial Expressions and Lower Bounds

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Show that BMA is strictly included in BR.

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Show a lower bound result on these combinatorial expressions.

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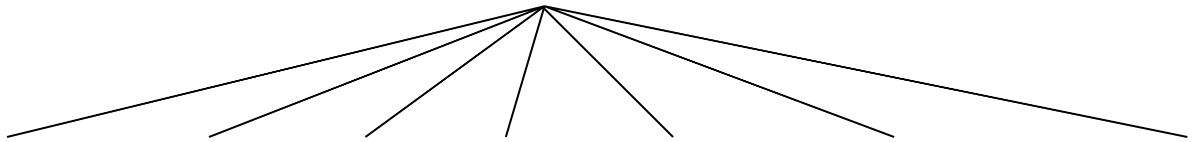

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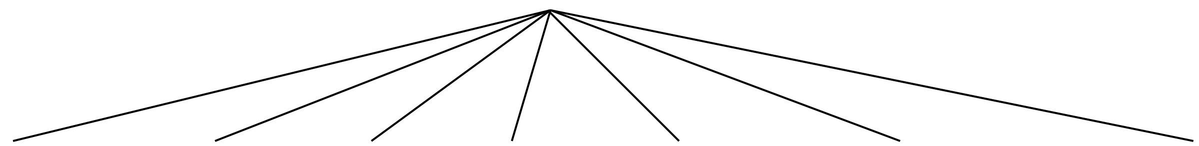

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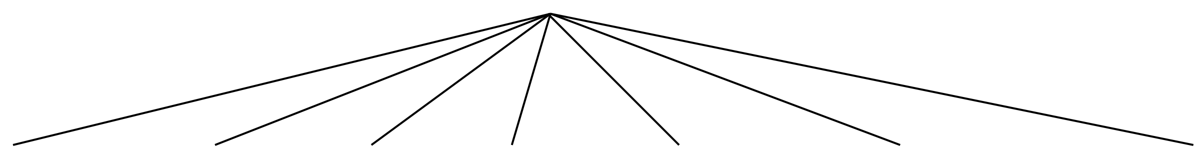

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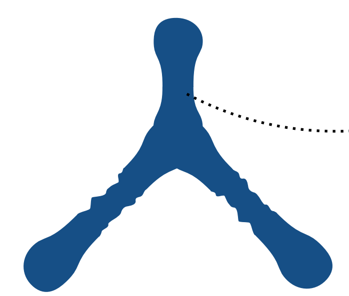

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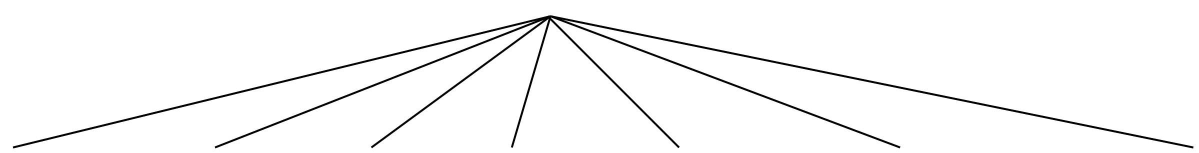

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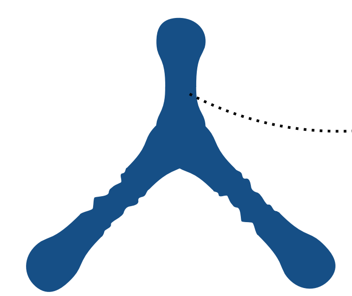

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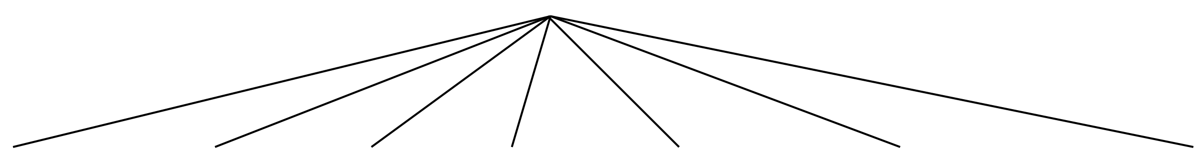

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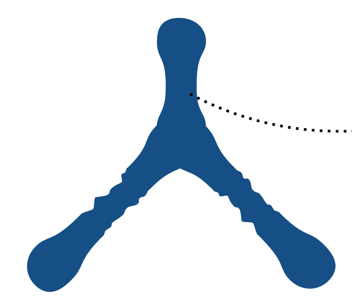

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Combinatorial expressions use such gates/functions and have bounded height (say, by h).

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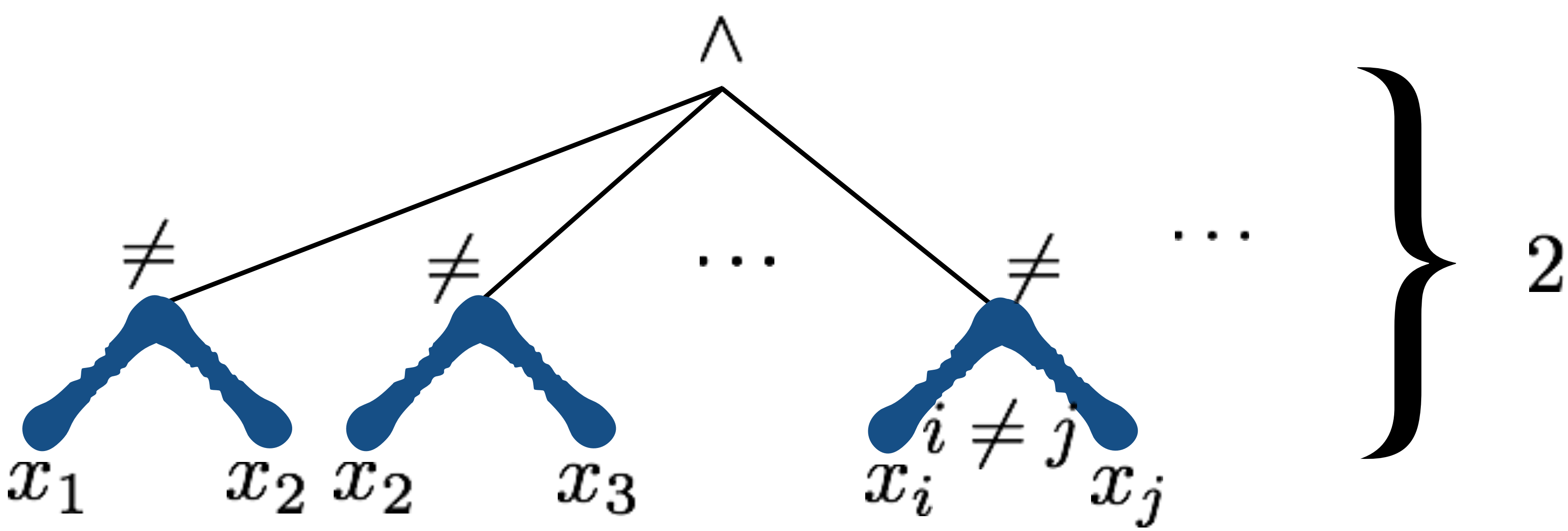

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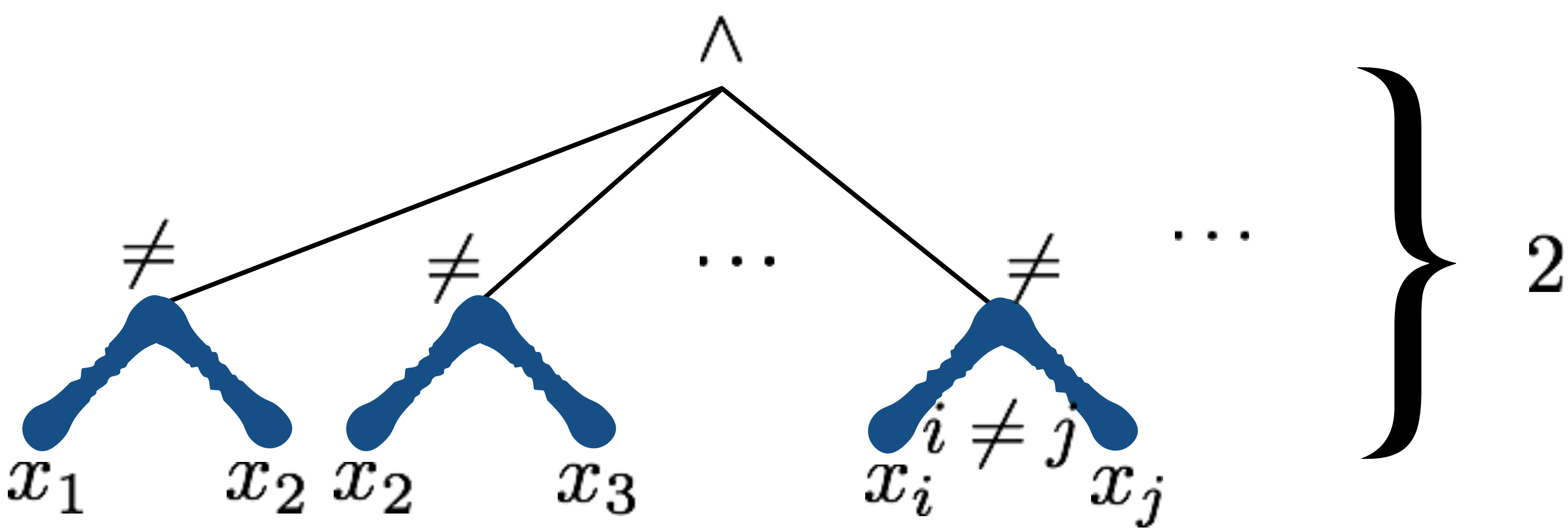
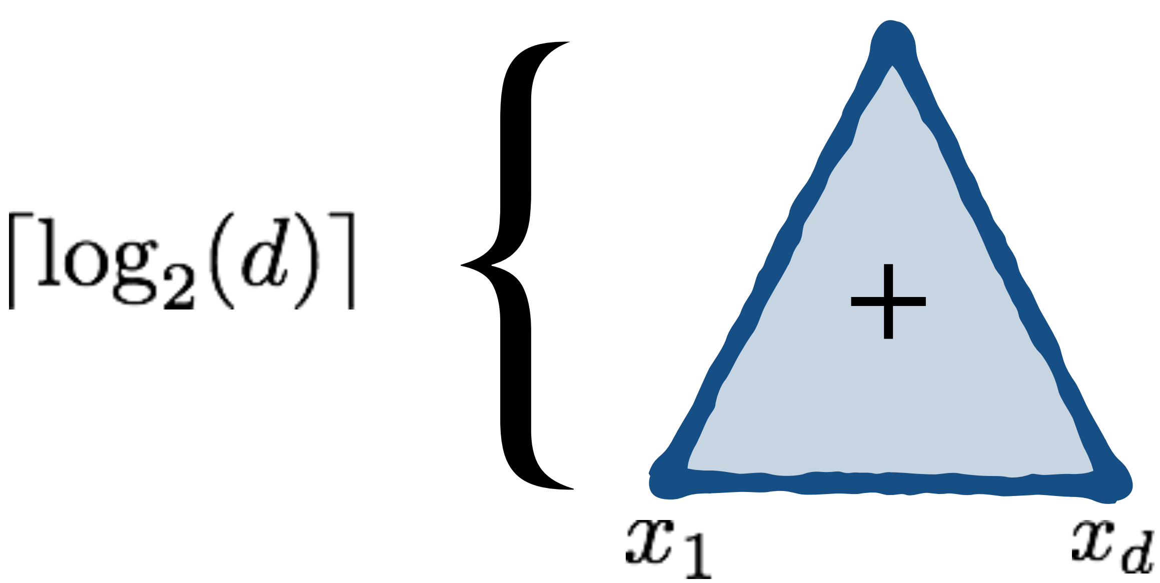


Sum



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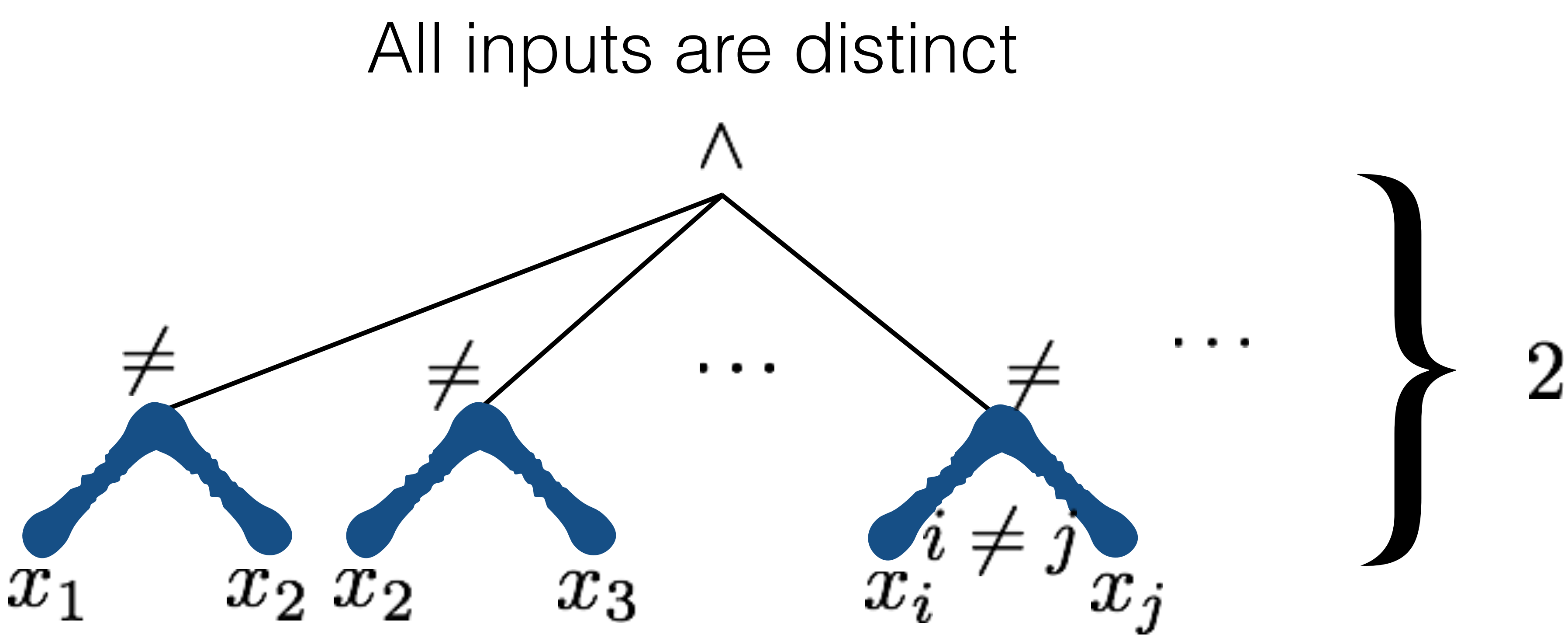
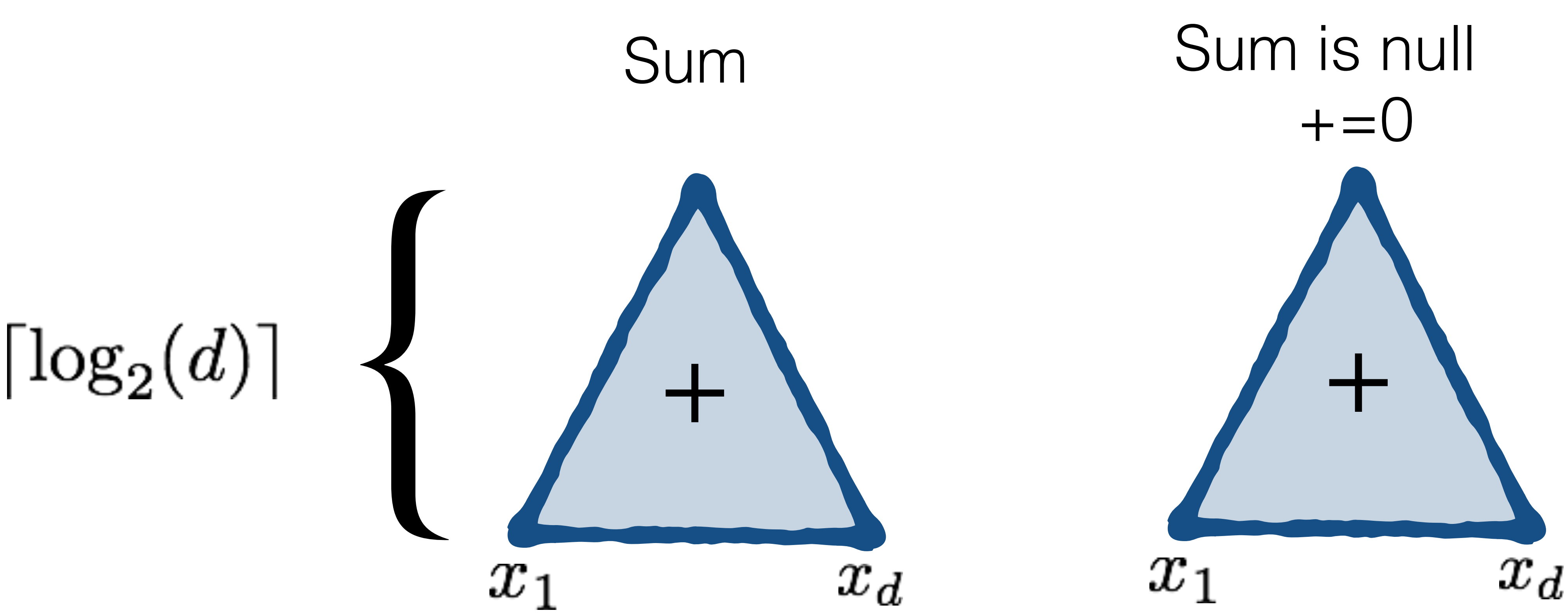

All inputs are distinct
$\wedge$
$\neq$
$\neq$
$\cdots$
$\neq$
$\cdots$
2
$i \neq j$
x_1
x_2
x_2
x_3
x_i
x_j


Sum
Sum is null
+=0
$\lceil \log_2(d) \rceil$
+
+
x_1
x_d
x_1
x_d

Normalization of expressions

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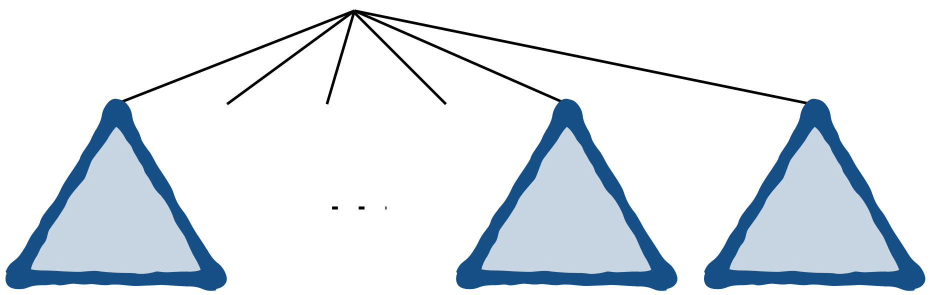
All expressions of height h and output in B can be transformed into a expressions of height h+1 and shape:



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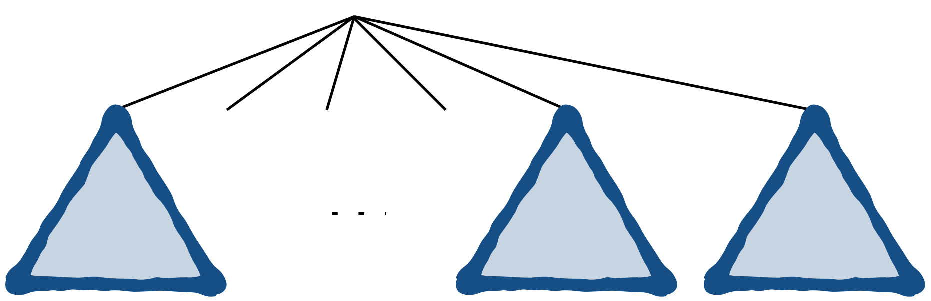

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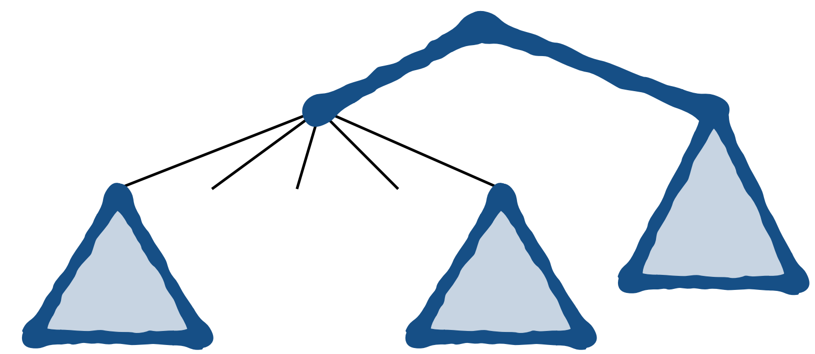

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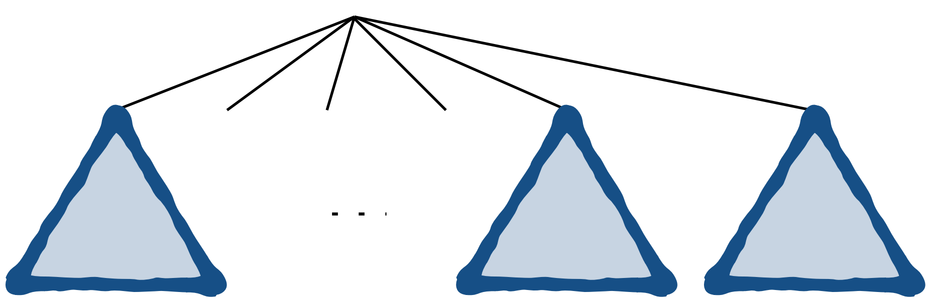

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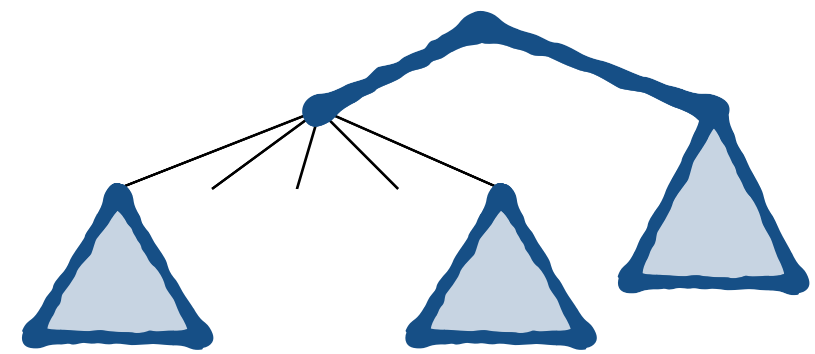

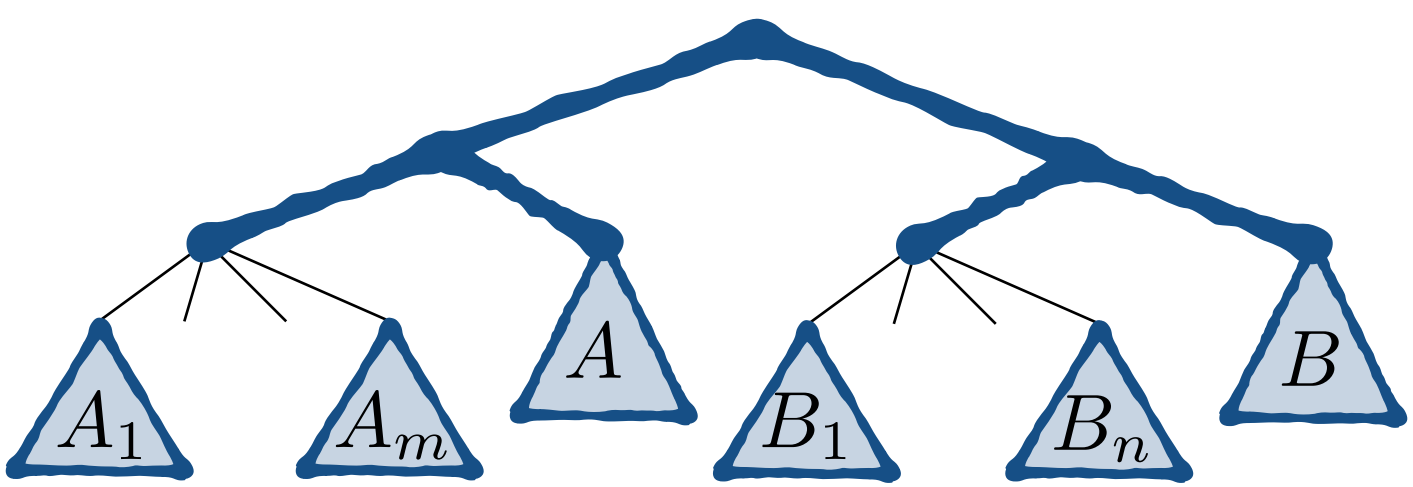

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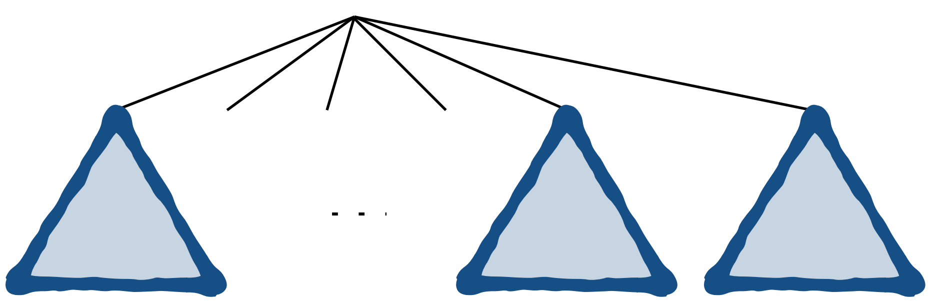

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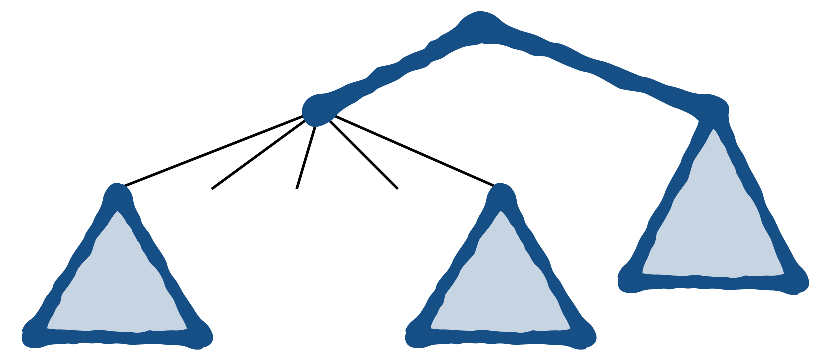

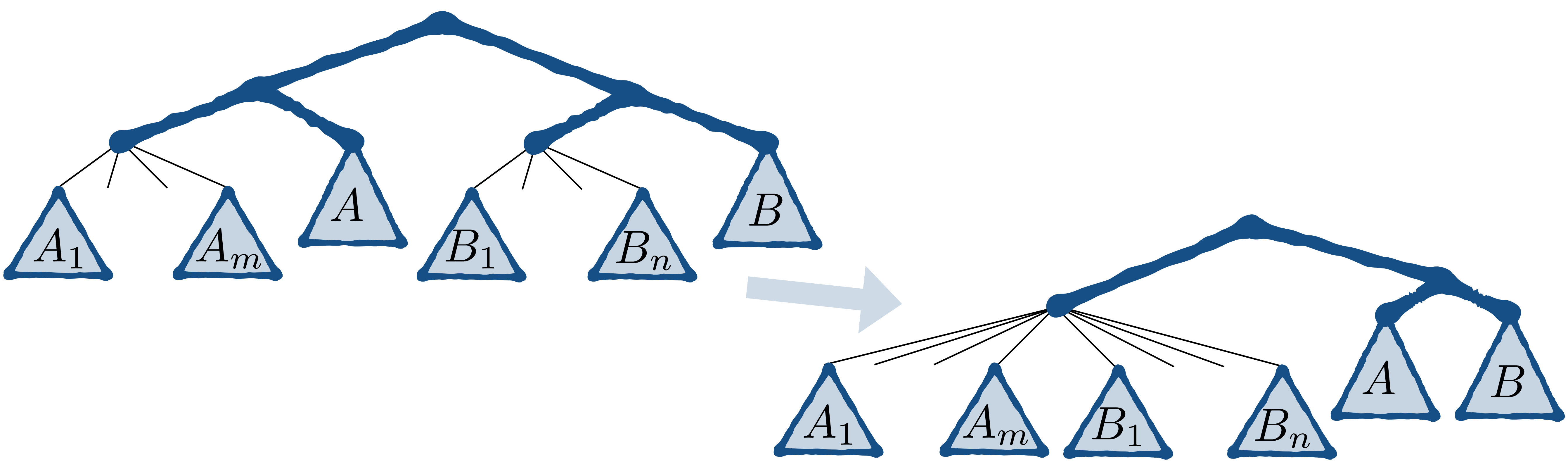

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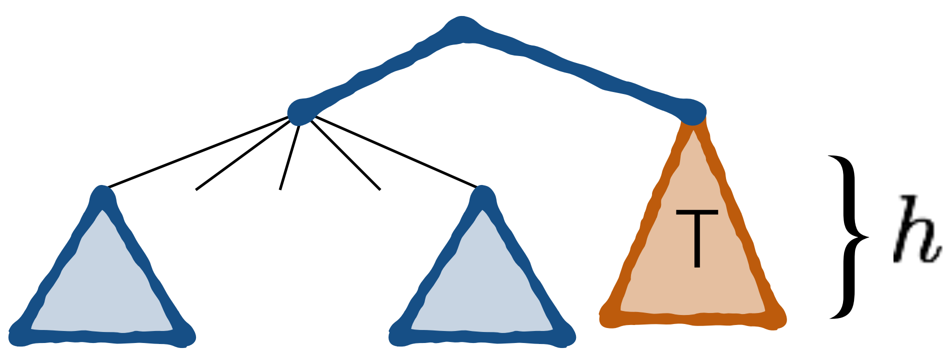

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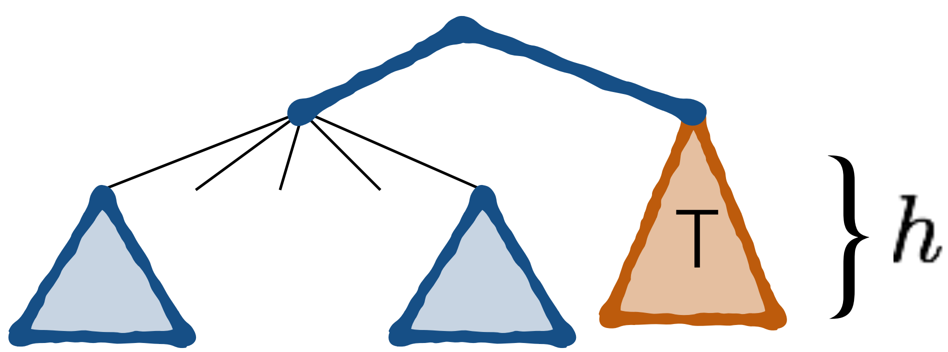

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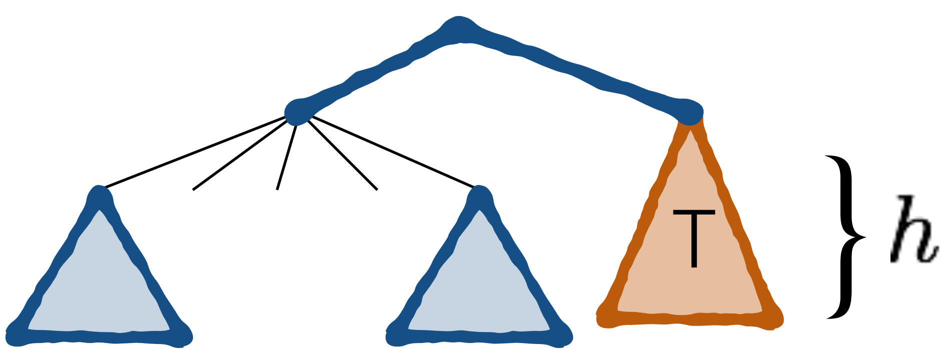

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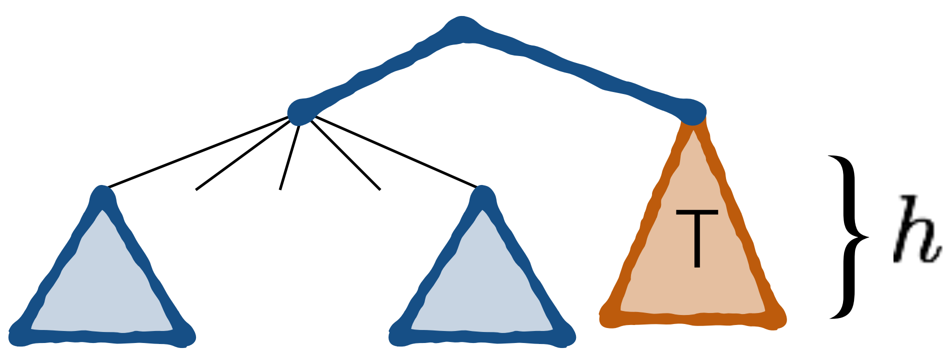

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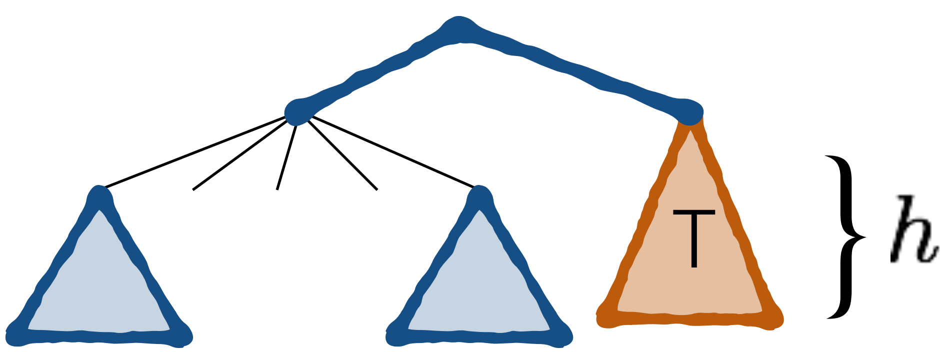

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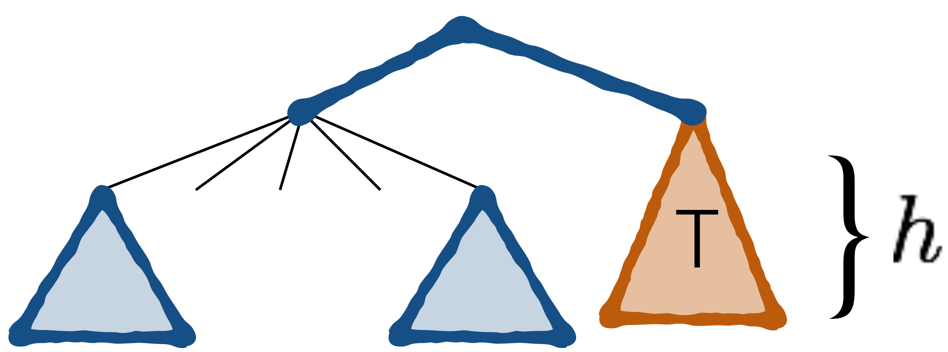

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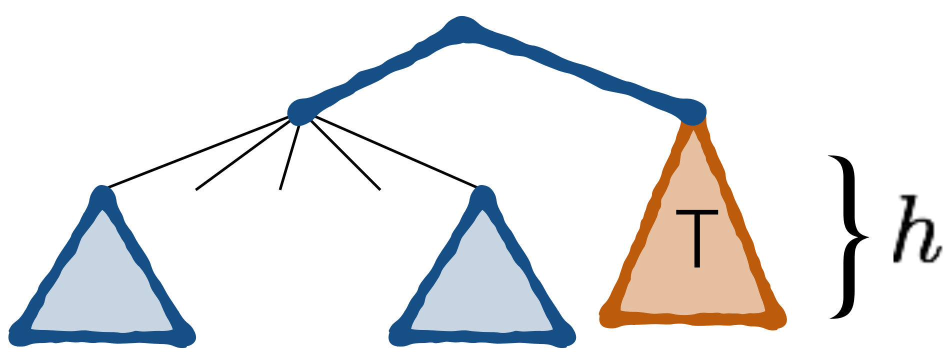

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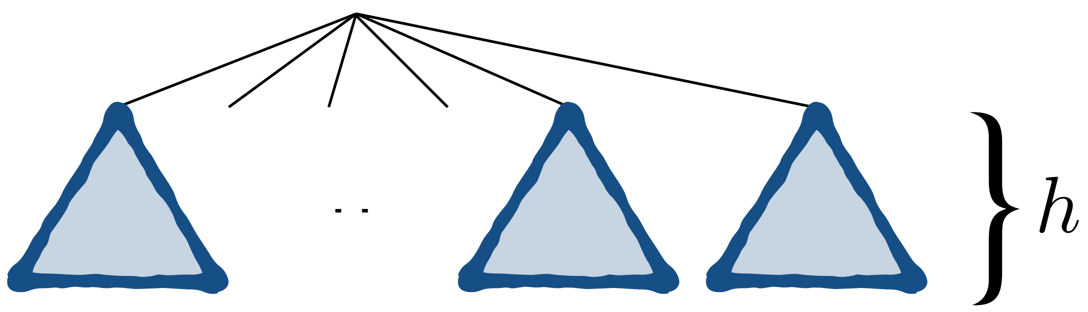

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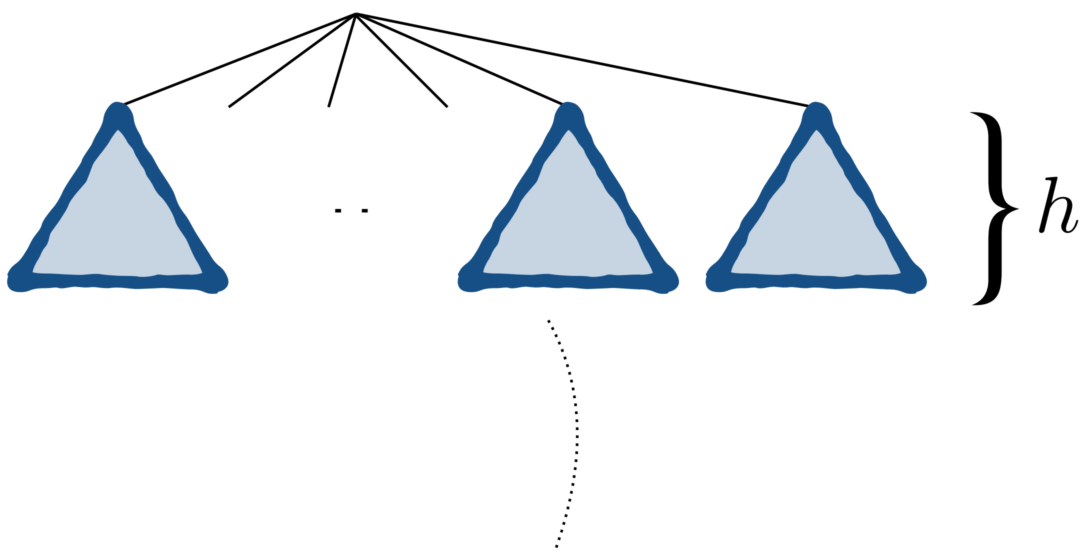

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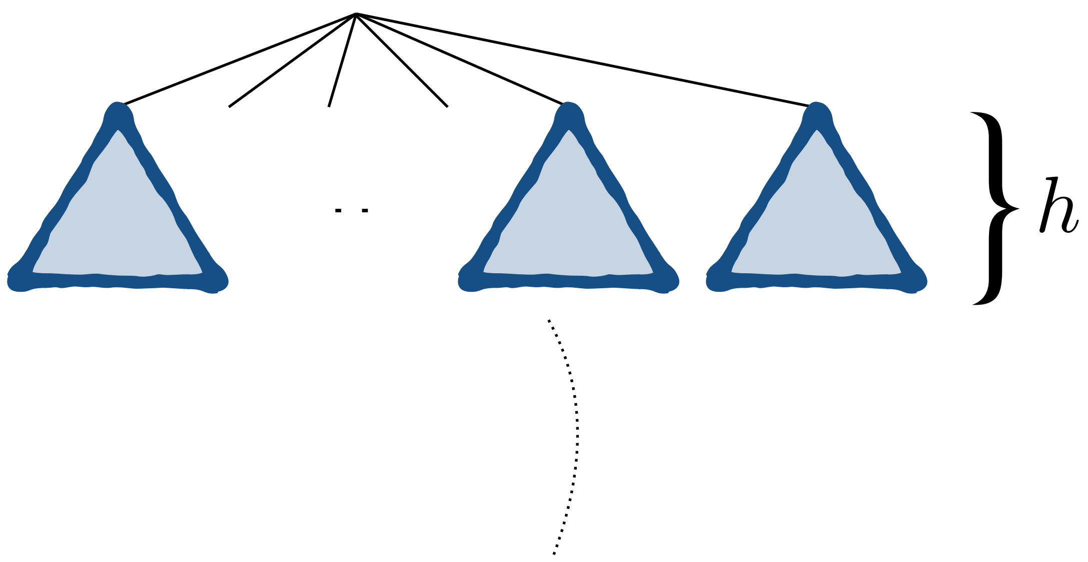

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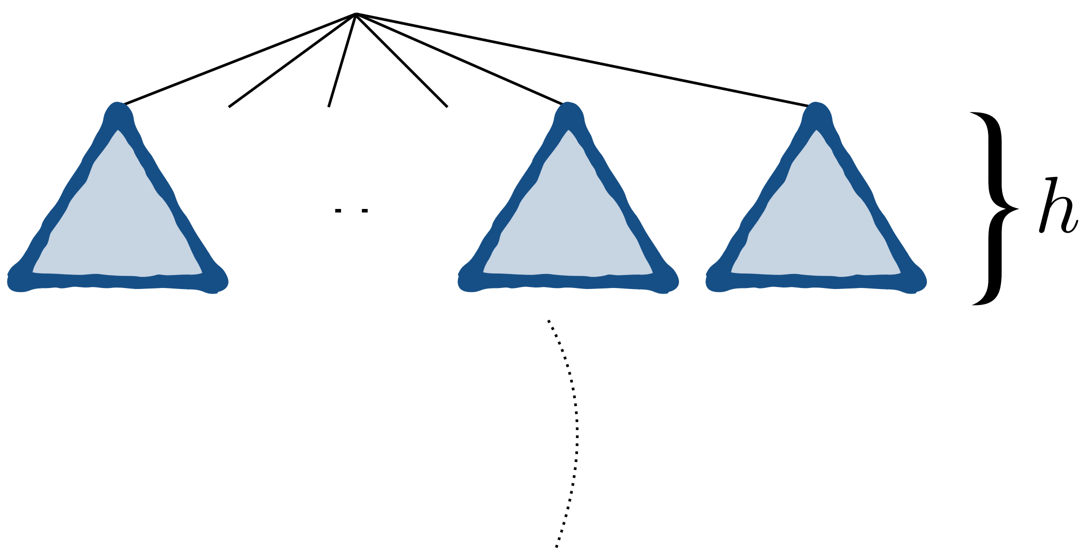

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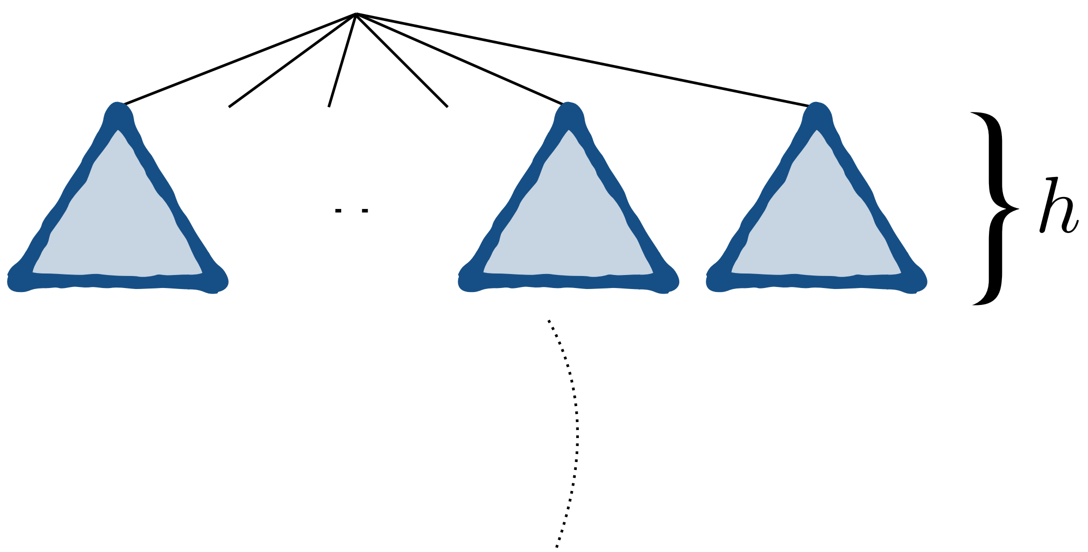

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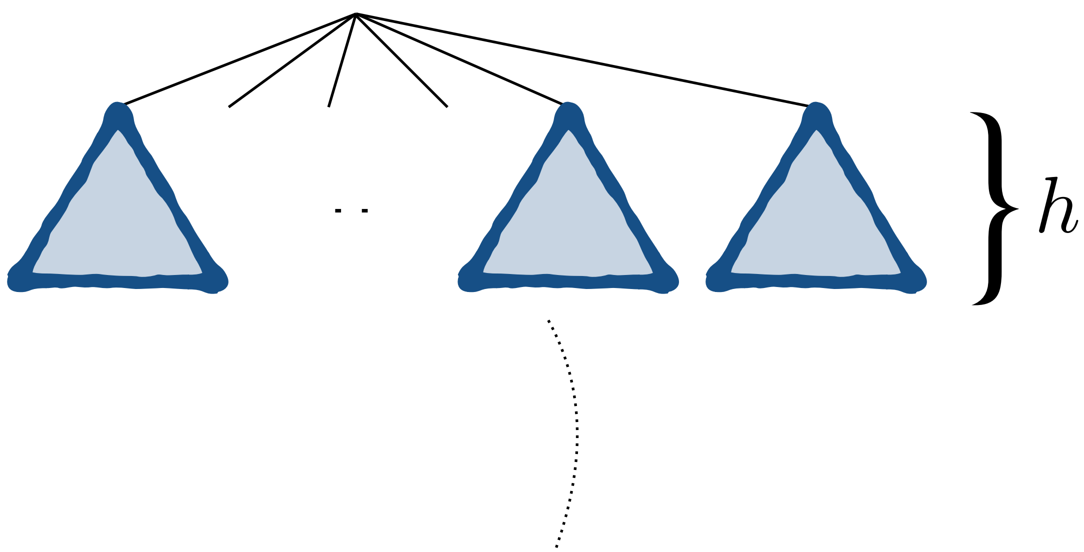

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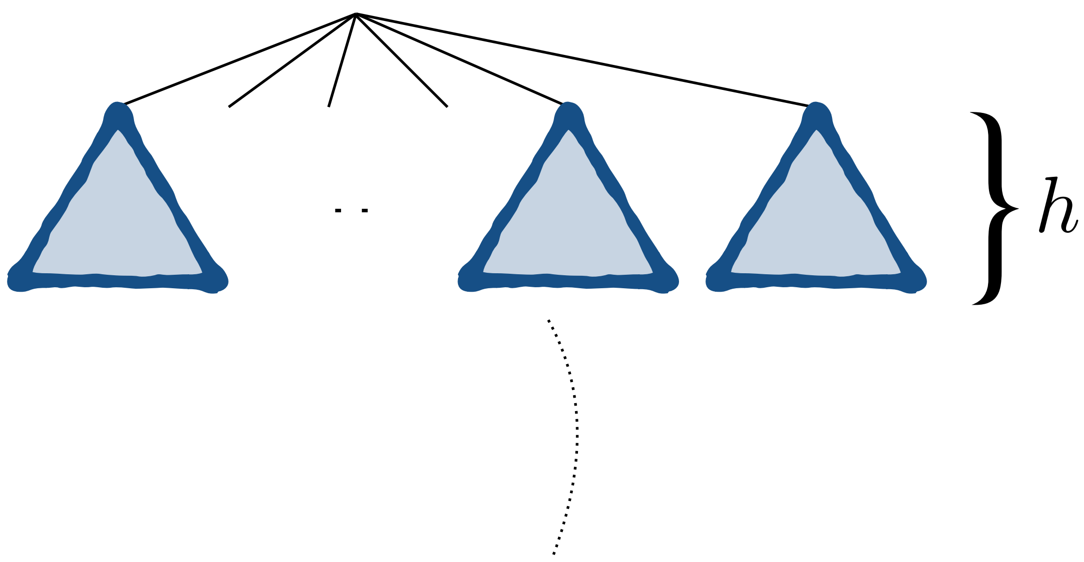

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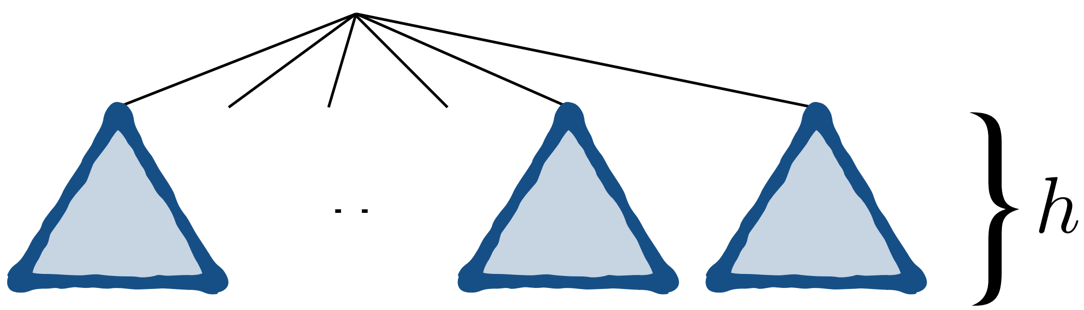

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Are the problems **sum=0** and **gcd=1** $\mathcal{W}$-definable for $\mathcal{W}$ non-trivial (*i.e.*, not containing the full window)?

Picture problems and reductions

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Theorem: A picture problem is $\mathcal{W}$-definable if and only if the line language L is $\mathcal{W}$-closed.

The lines that resemble a line from L through any window, belong to L.

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The lines that resemble a line from L through any window, belong to L.

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Not closed!

$L = \{u \in \{0,1\}^d \text{ that contains a '0'}\}$

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This shows that the **gcd=1** problem is at least as hard as the picture problem 'all lines contain a 0'.

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Close to the proof in:
[Pascal Tesson. An application of the Hales-Jewett theorem to multiparty communication complexity. Extract from the PhD Thesis, 2004]

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A selection gate computes

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Finite variants:
As usual if the domain D is finite, but sufficiently large, similar results holds (compactness):

- Fix B to be {0,1}. For all h and all s, there exists n such that, **sum=0 mod n** over h inputs ranging over [0,n-1] is not doable by a formula of height at most h and size at most s.

Conclusion

Applications:

- these expressions are motivated for logic separation results
  - a toy example is present in the paper (metafinite structures)
  - a more difficult example is the BMA - BR separation,
  - others ?

Thank you!