Self-Organizing Flows in Social Networks

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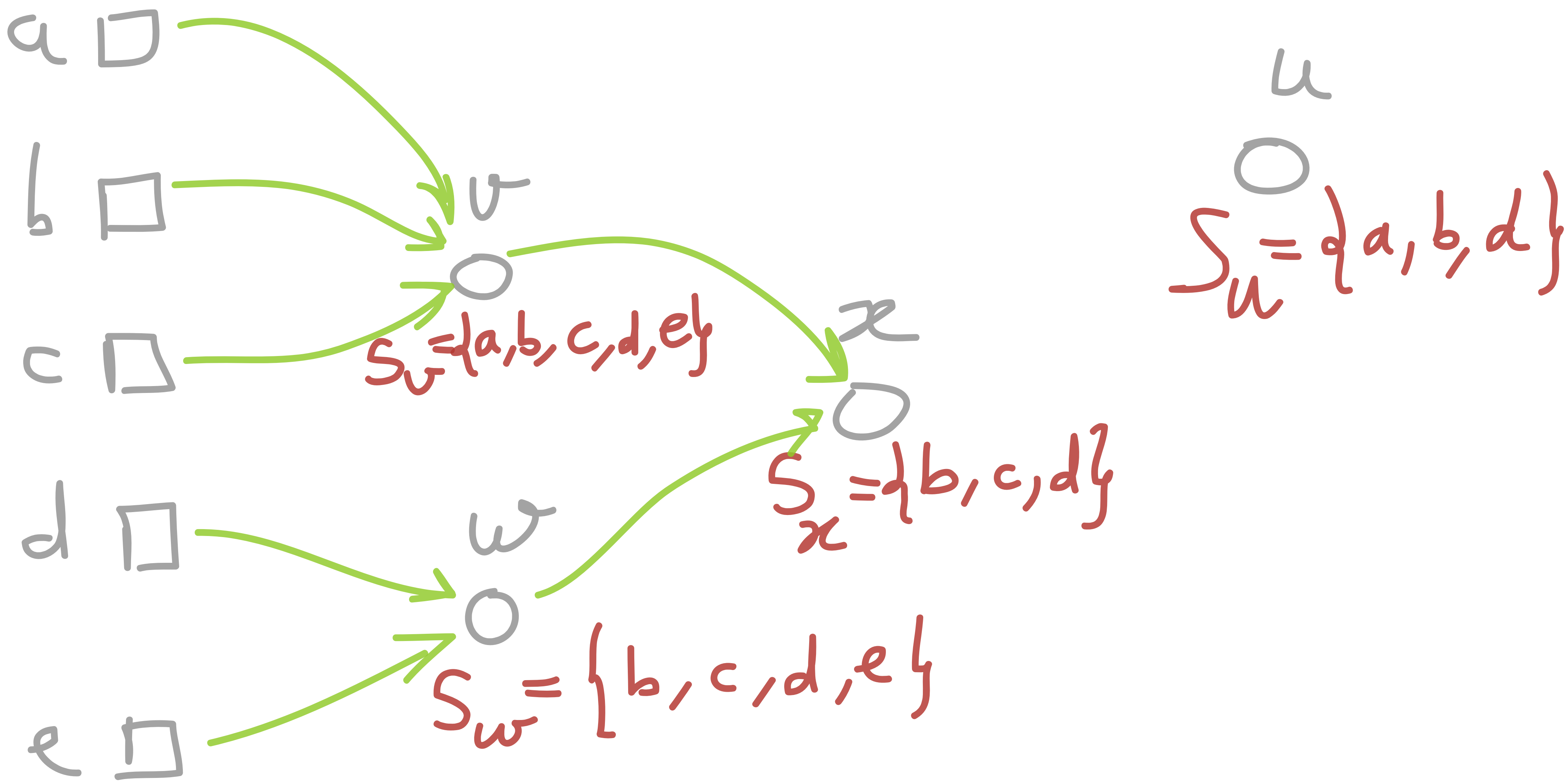
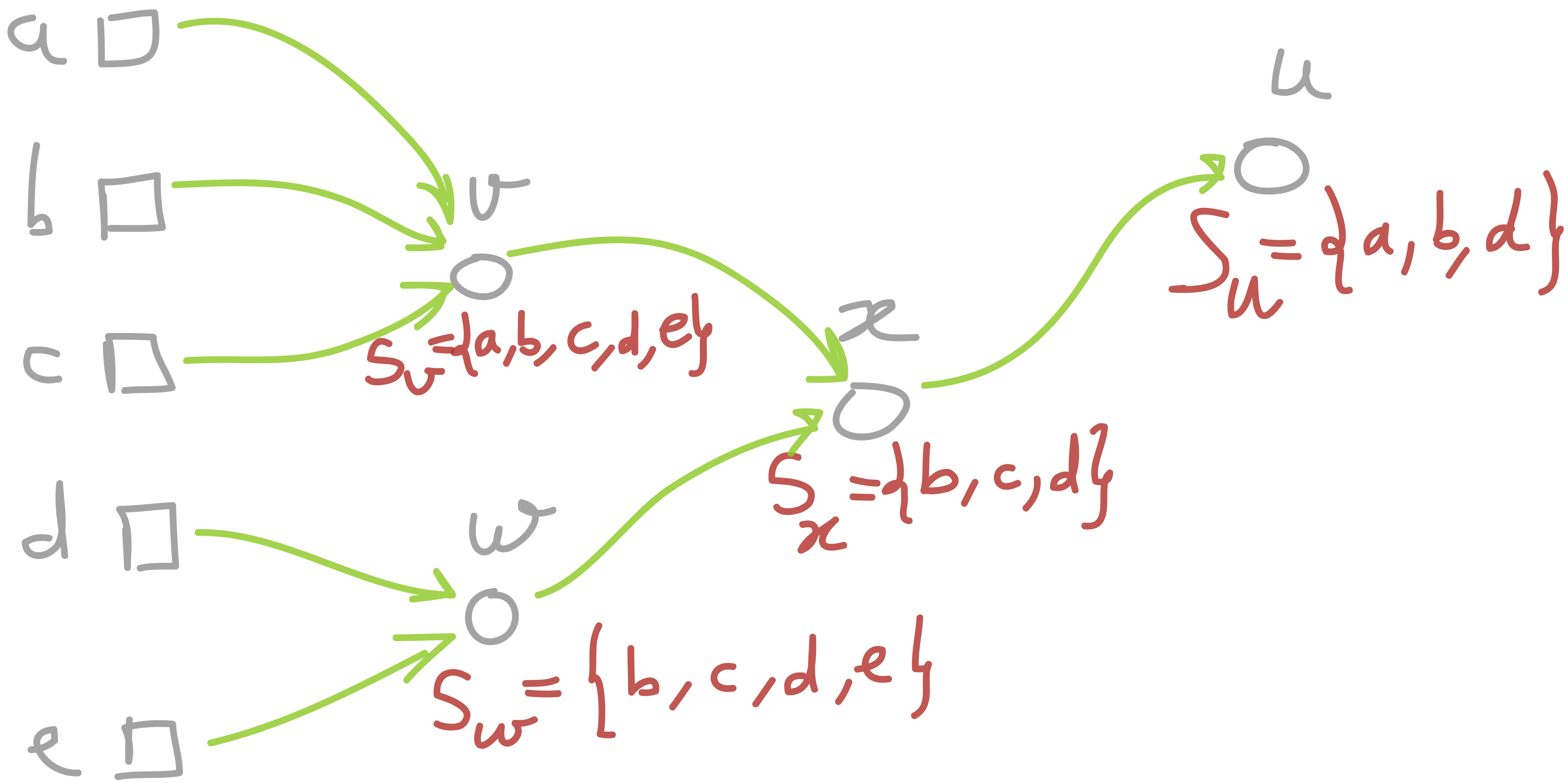
Network flow game

- Twitter like game:
  - To play: change your connections
  - The goal: gather interesting information
  - The cost: filter out spam

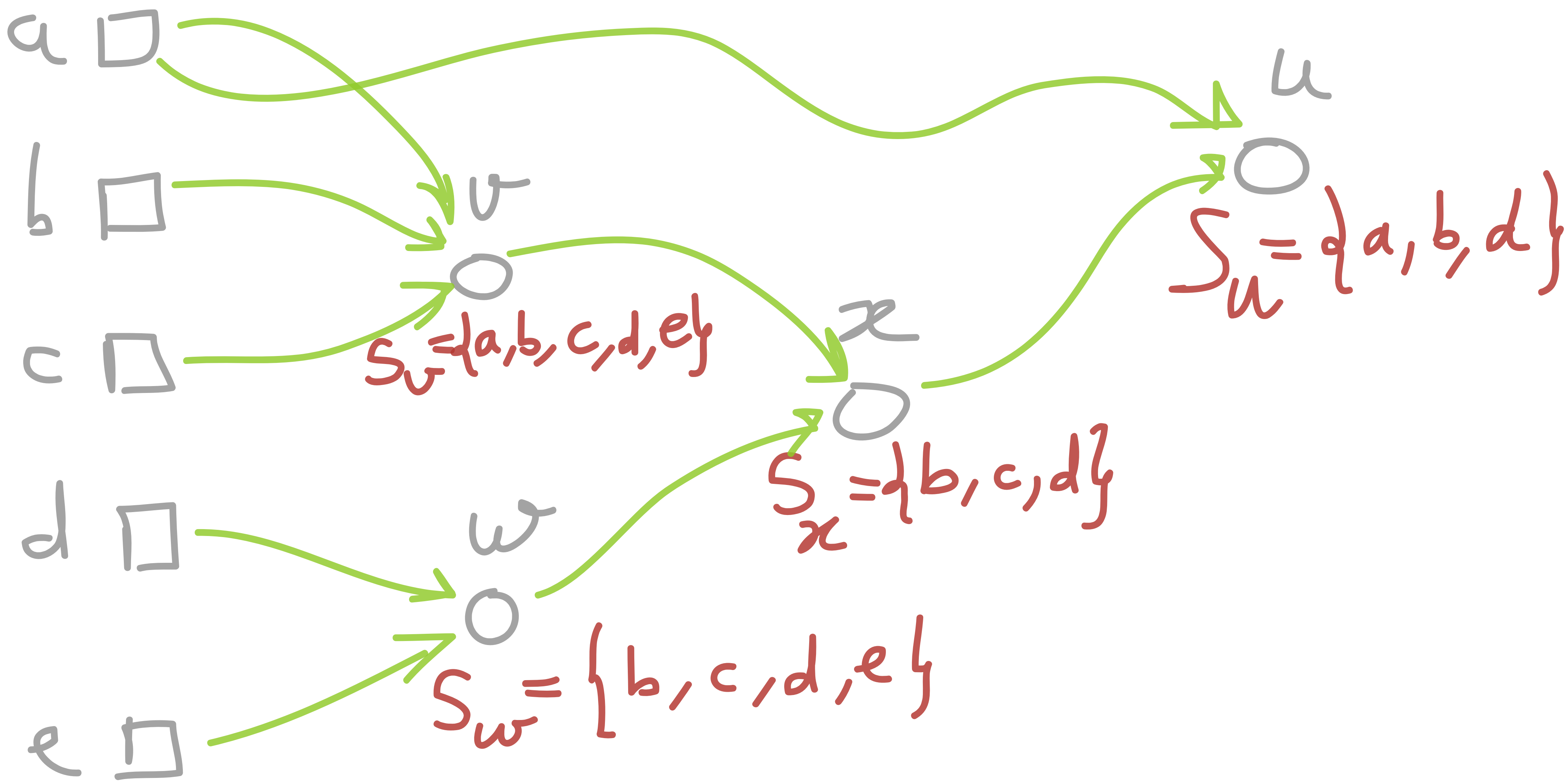
Network flow game



Network flow game



Network flow game



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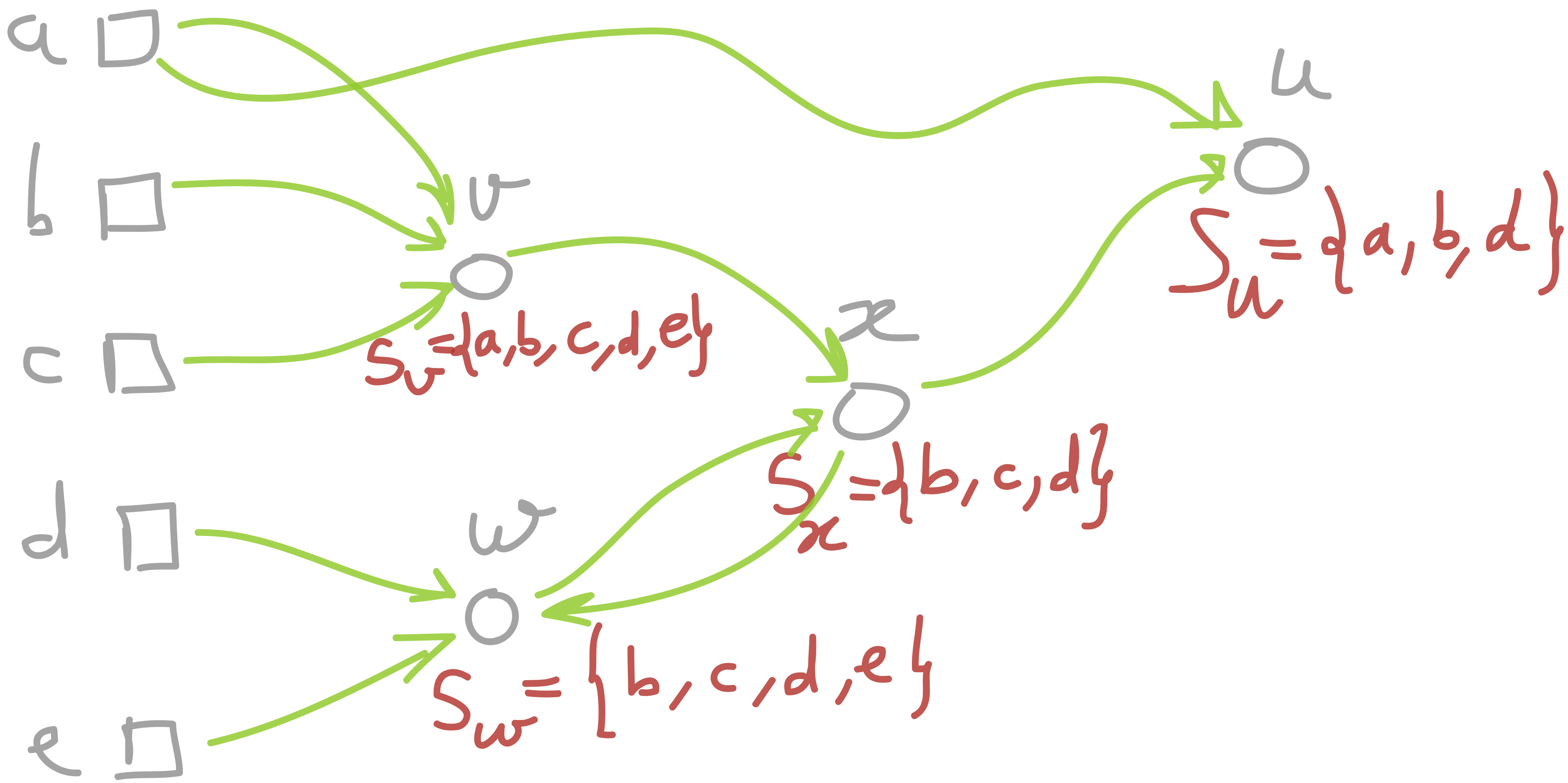
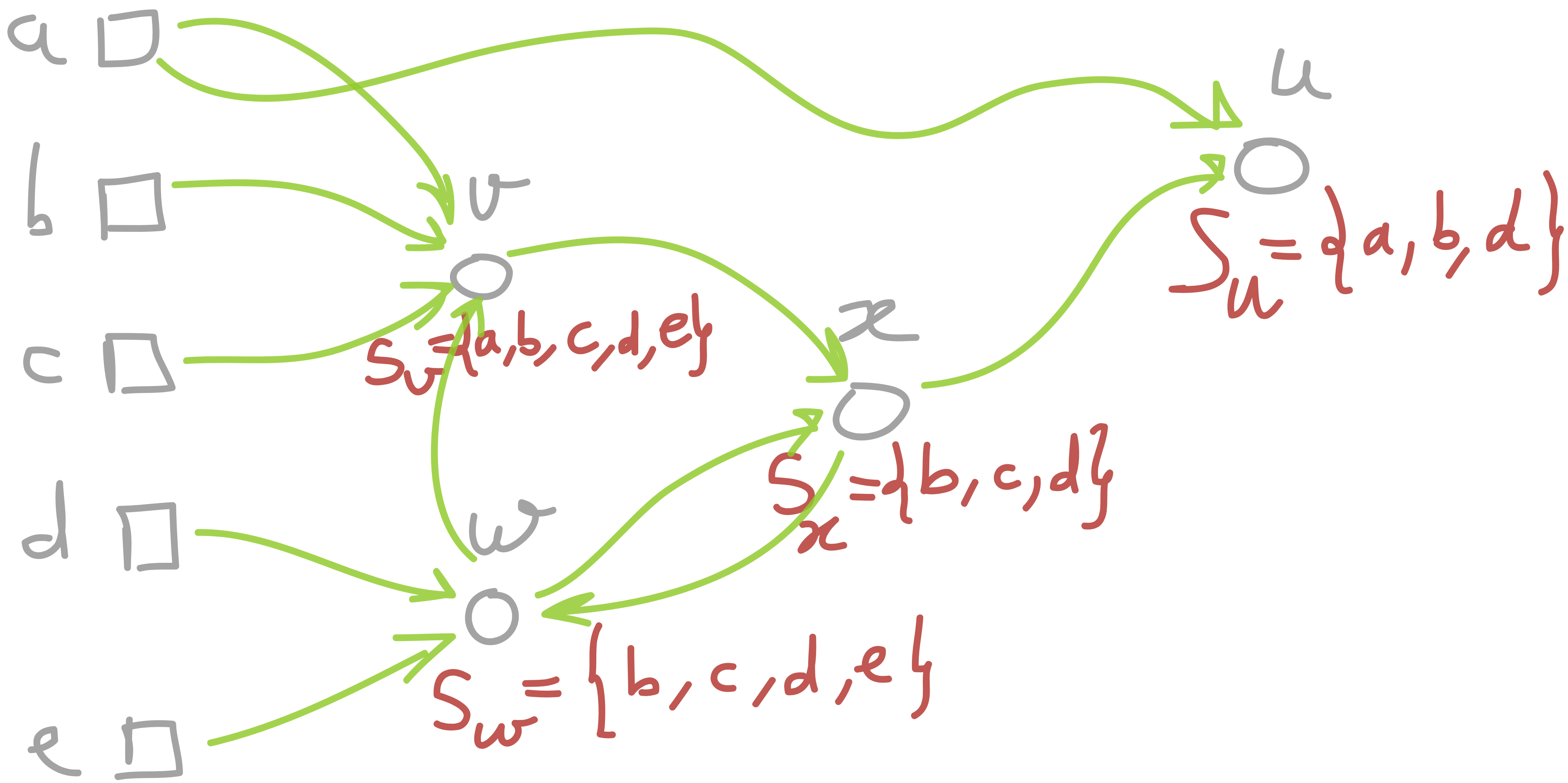
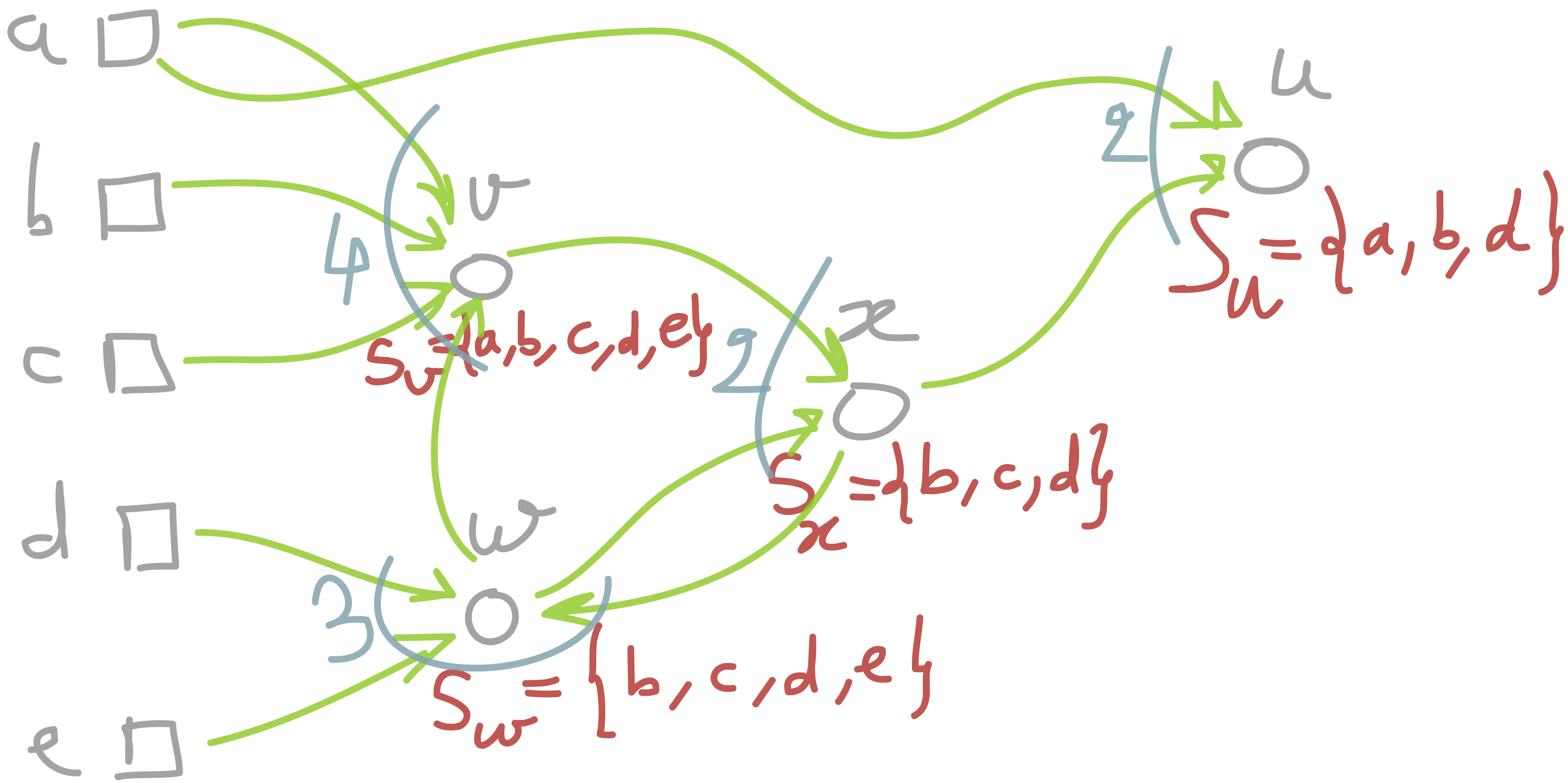


Network flow game



Network flow game



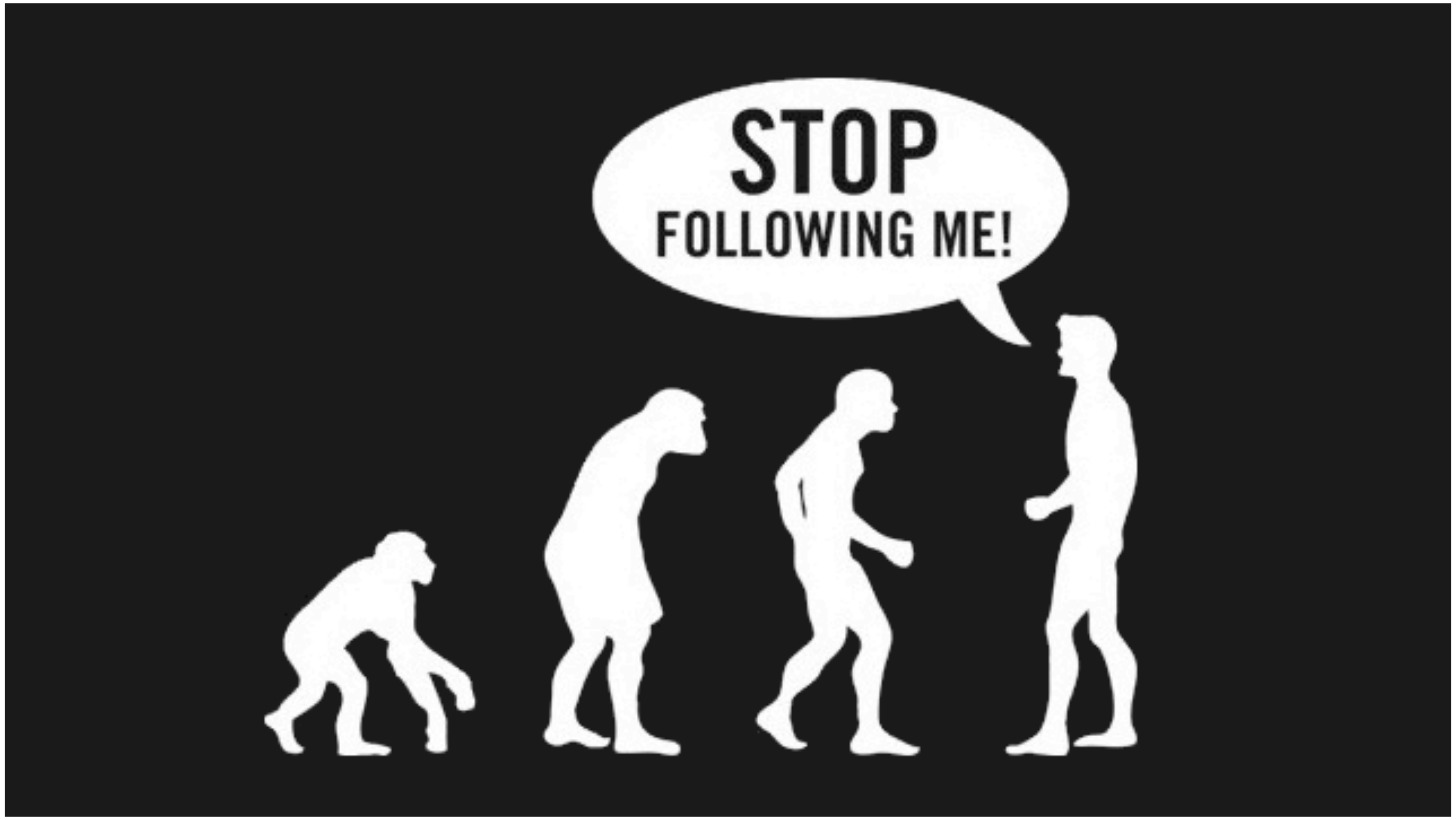
Model

- Interests :
  - Each user u has an interest set $S_u \subseteq S$.
  - She retransmits news about subjects in S_u.
- Links :
  - User u can create link vu (u « follows » v).
- Budget of attention :
  - User u can follow at most D_u nodes.

Problem

- Who should I follow ?

Problem



Problem

- Each user u is a player of the following game :
  - change the users she follows (with deg ≤ D_u)
  - to maximize $U_u = |R(u) \cap S_u|$ where R(u) denotes the subjects she receives.
- How does this evolves (selfish dynamics) ?

Questions to answer

- Does this converges ?
- If so, what is the price of anarchy :
  - U*=∑U_u under best global choice of links,
  - over U=∑U_u under worse selfish equilibrium.

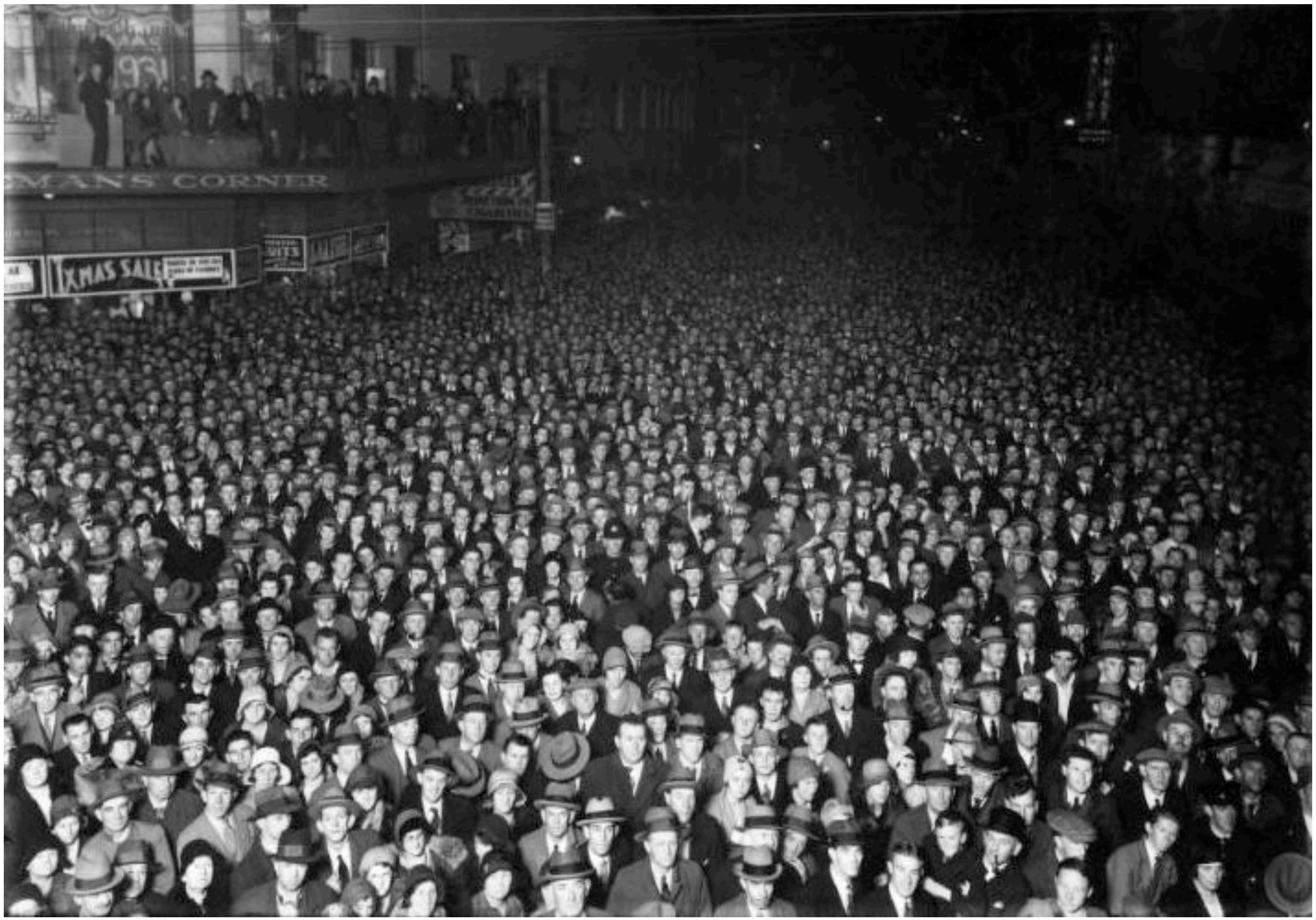
Related work

- Convergence of dynamics [Rosenthal '73, Monderer & Shapley '96]
- Network creation games [Roughgarden '07, ...] (connectivity, distances, influence, ...)
- B-matching and preferences in P2P [Mathieu et al. '07]
- Communities as a coloring game [Kleinberg &Ligett'10] [Ducoffe, Mazauric, Chaintreau'13]

Outline

- Homogeneous interests
- Heterogeneous interests
- Metric model of interests

Homogeneous interests

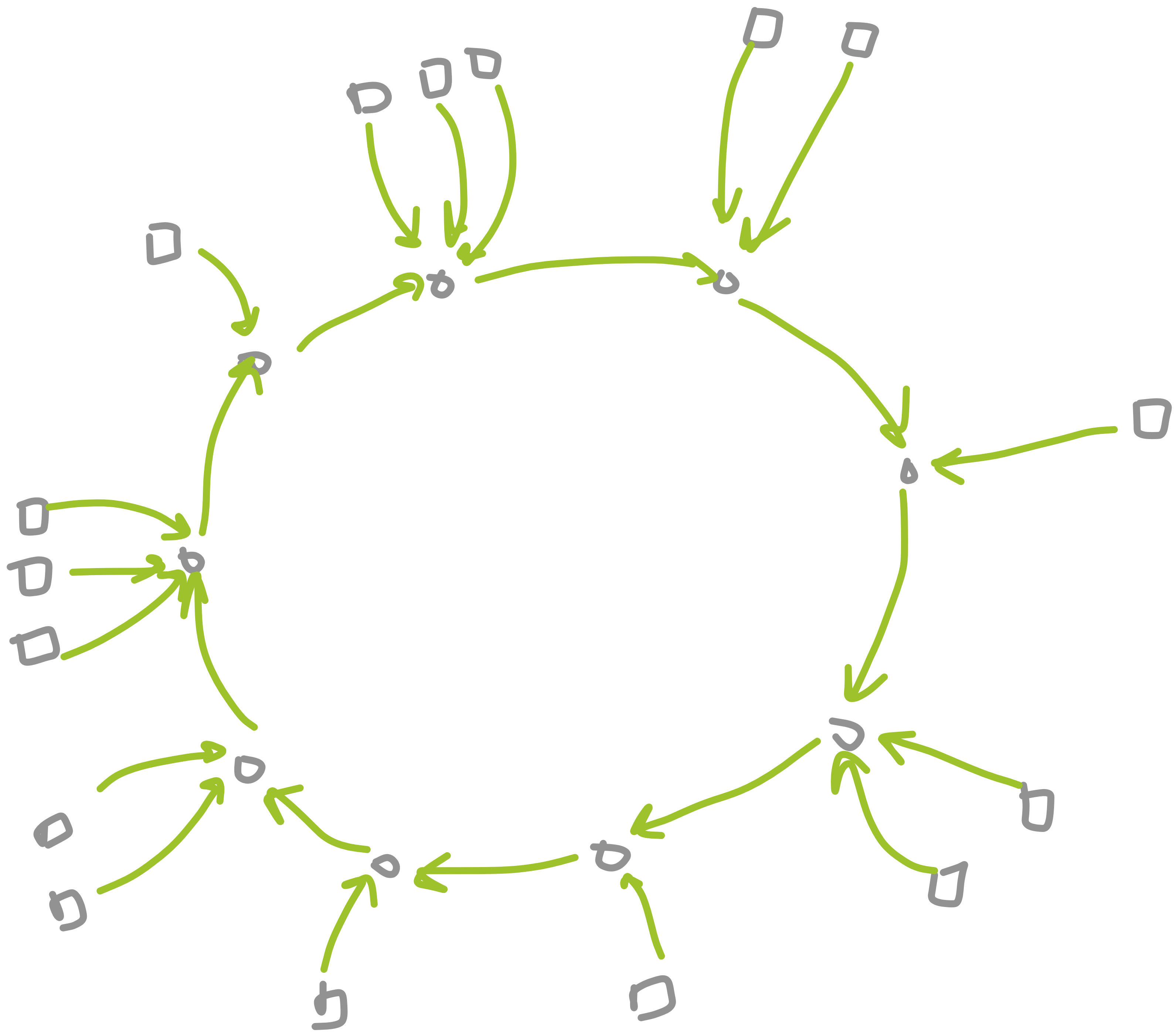


Homogeneous interests

- Assume all nodes have same interest set S.
- Def : U*is the highest utility a node can get.
- Th 1 : If D_u ≥ 3 for all u, then selfish dynamics always converge to a Nash equilibrium where each user receives at least (d-2)/(d-1) U* subjects.
- The price of anarchy is thus 1+O(1/d).

Proof idea

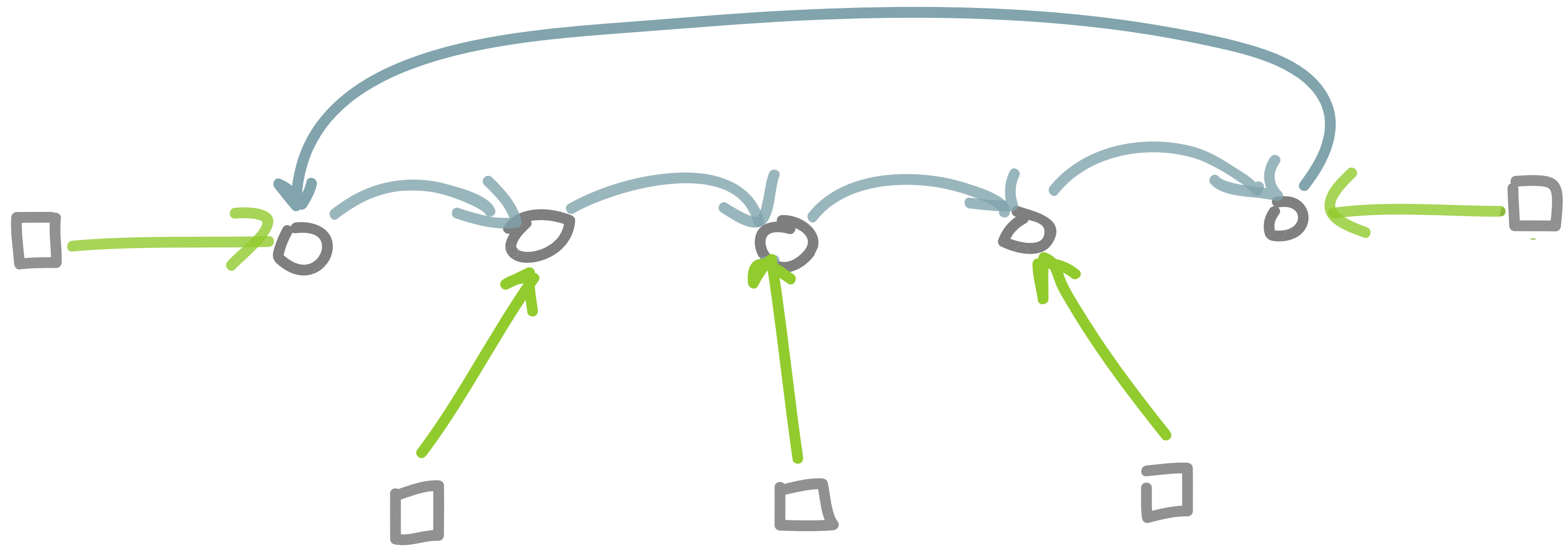
- Stable solution is not too far from optimal



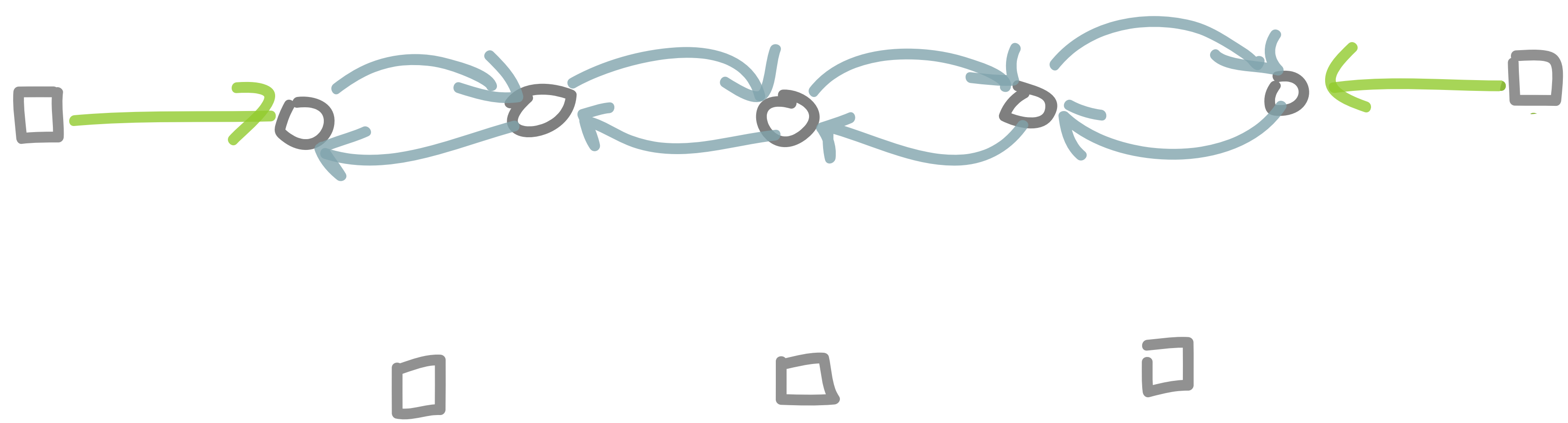
Proof idea

- $D_u \geq 3$ implies strong connectivity
- No transitivity arc implies $m \leq 2n$
- At most 2 links per node for connectivity
- d-2 links for gathering subjects instead of d-1

Degree 2



(a) Benchmark configuration

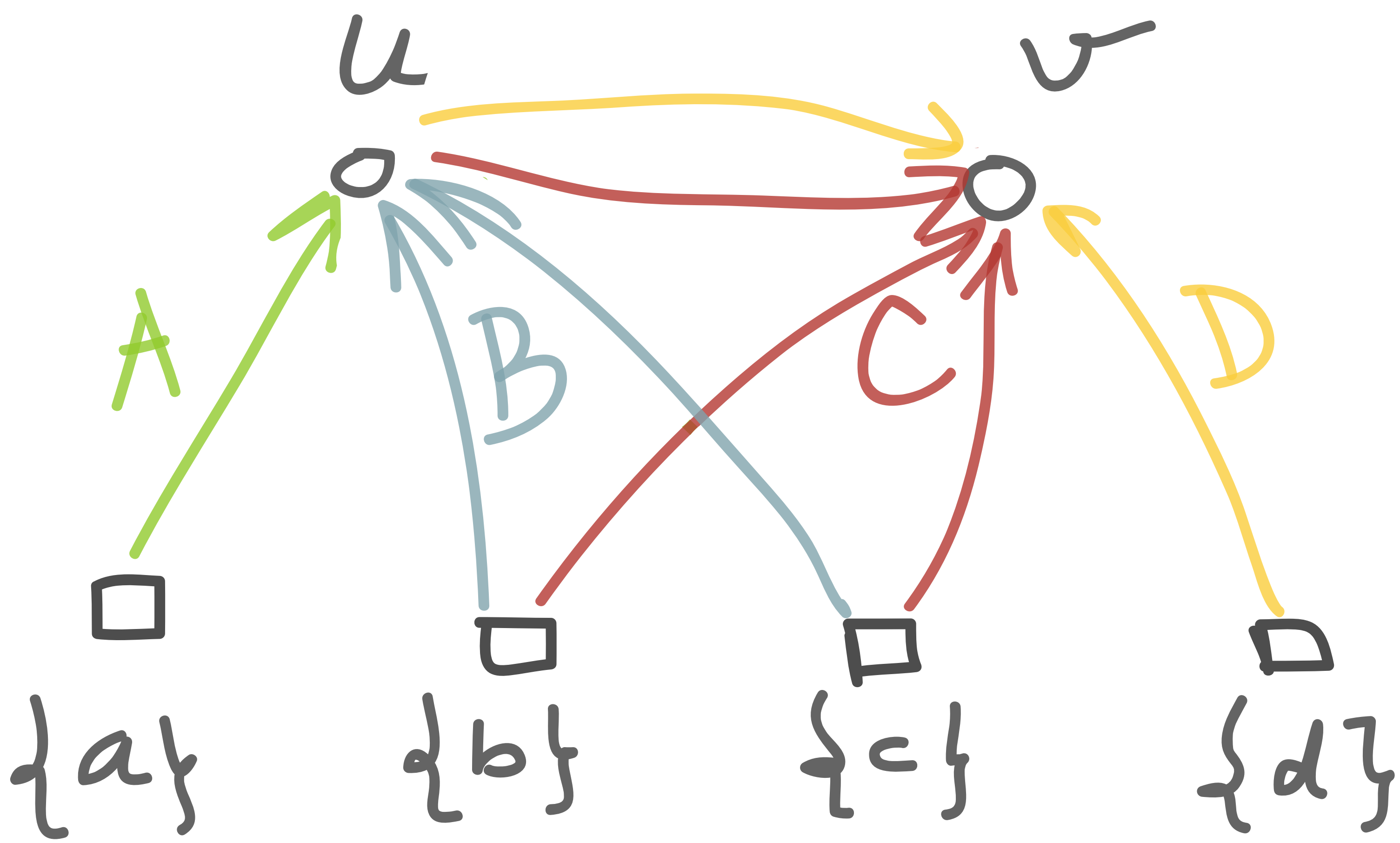


(b) A Nash equilibrium configuration

Proof idea : dynamics

- n_i = number of users gathering i subjects
- $(n_0, n_1, \ldots, n_p)$ decreases in lexicographic order
- $- \sum n_i n^{p-i}$ is a potential function.

Not a congestion game



A 4-cycle $(A, C) \rightarrow (B, C) \rightarrow (B, D) \rightarrow (A, D) \rightarrow (A, C)$ in the strategy space.

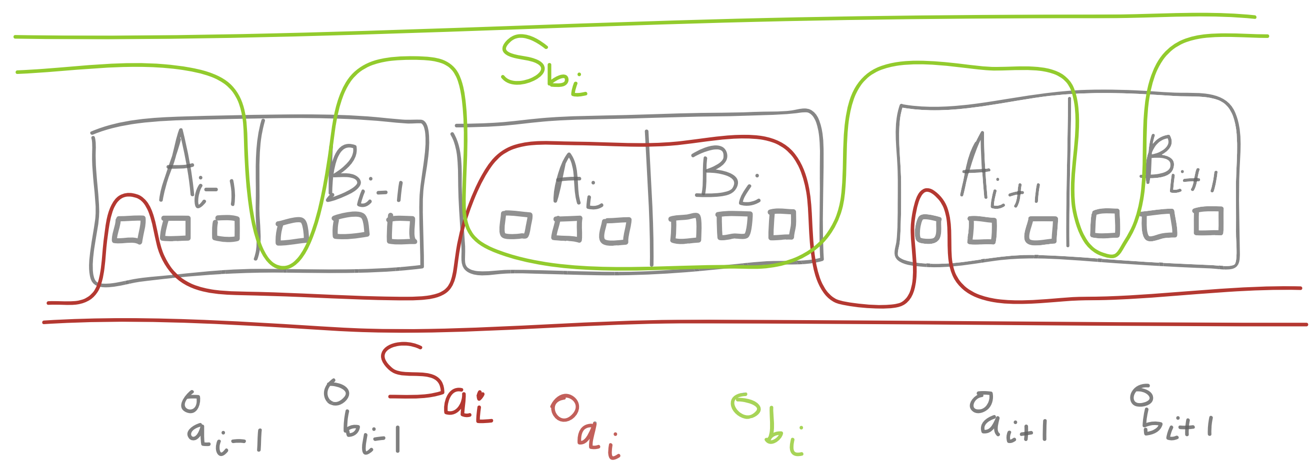
+1 +1 -1 +2

Heterogeneous interests

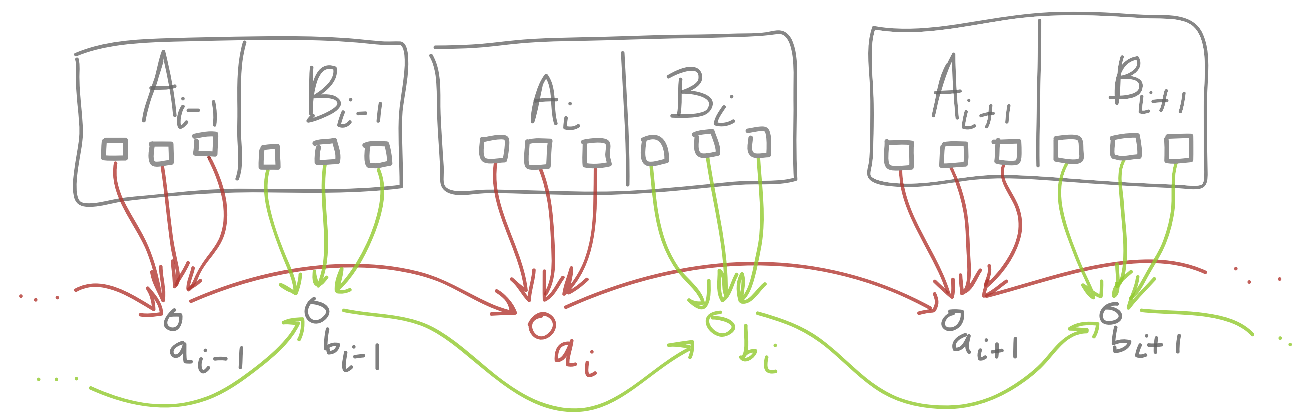
Heterogeneous interests

- Th 3 : The price of anarchy can be $\Omega(n/d)$.
- Prop : Selfish user dynamics may not converge.

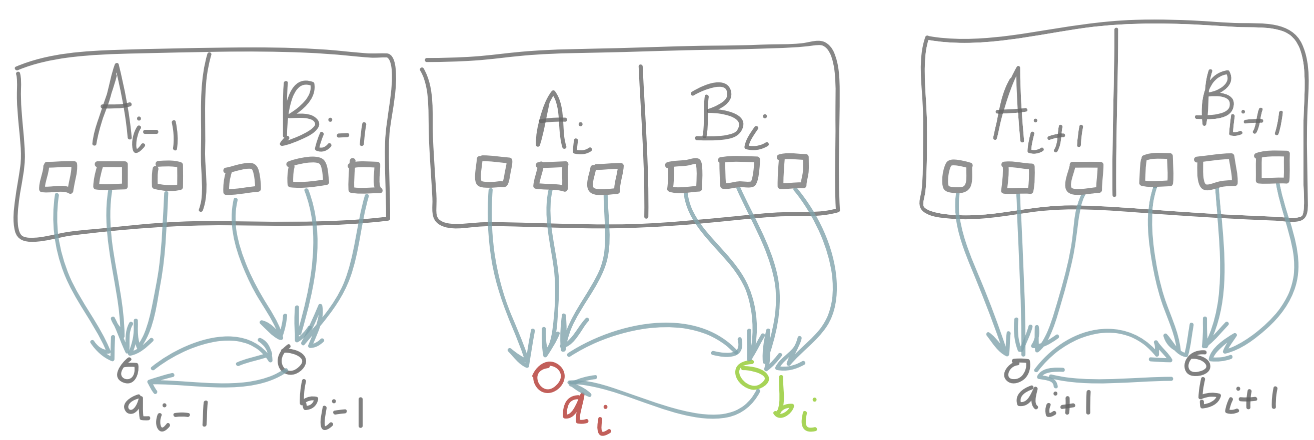
Price of Anarchy



(a) Interest sets



(b) Benchmark configuration



(c) A Nash equilibrium configuration

Non convergence

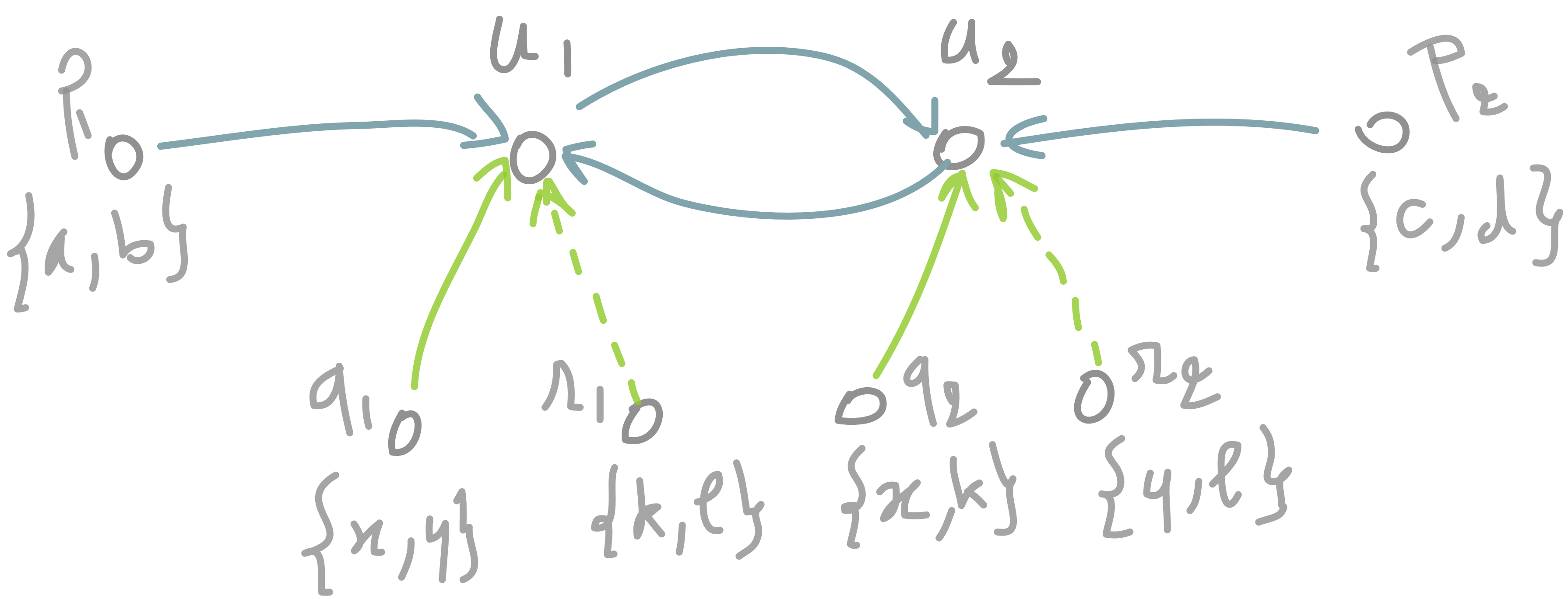


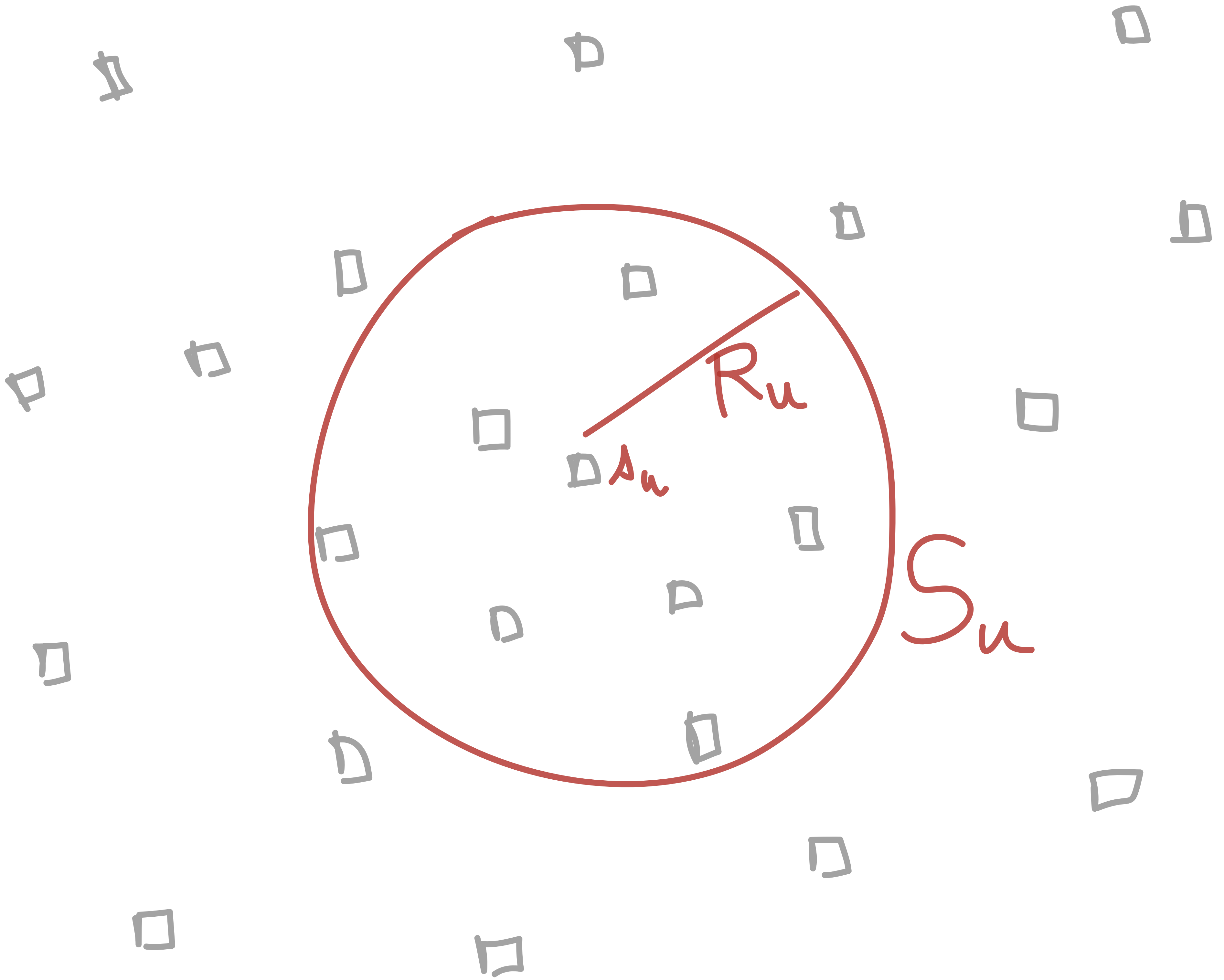
Figure 4: Instability with heterogeneous interest sets.

User\Topic	a	b	c	d	x	y	k	l
u_1	2	2	2	0	ϵ	1	1	ϵ
u_2	2	0	2	2	1	ϵ	ϵ	1

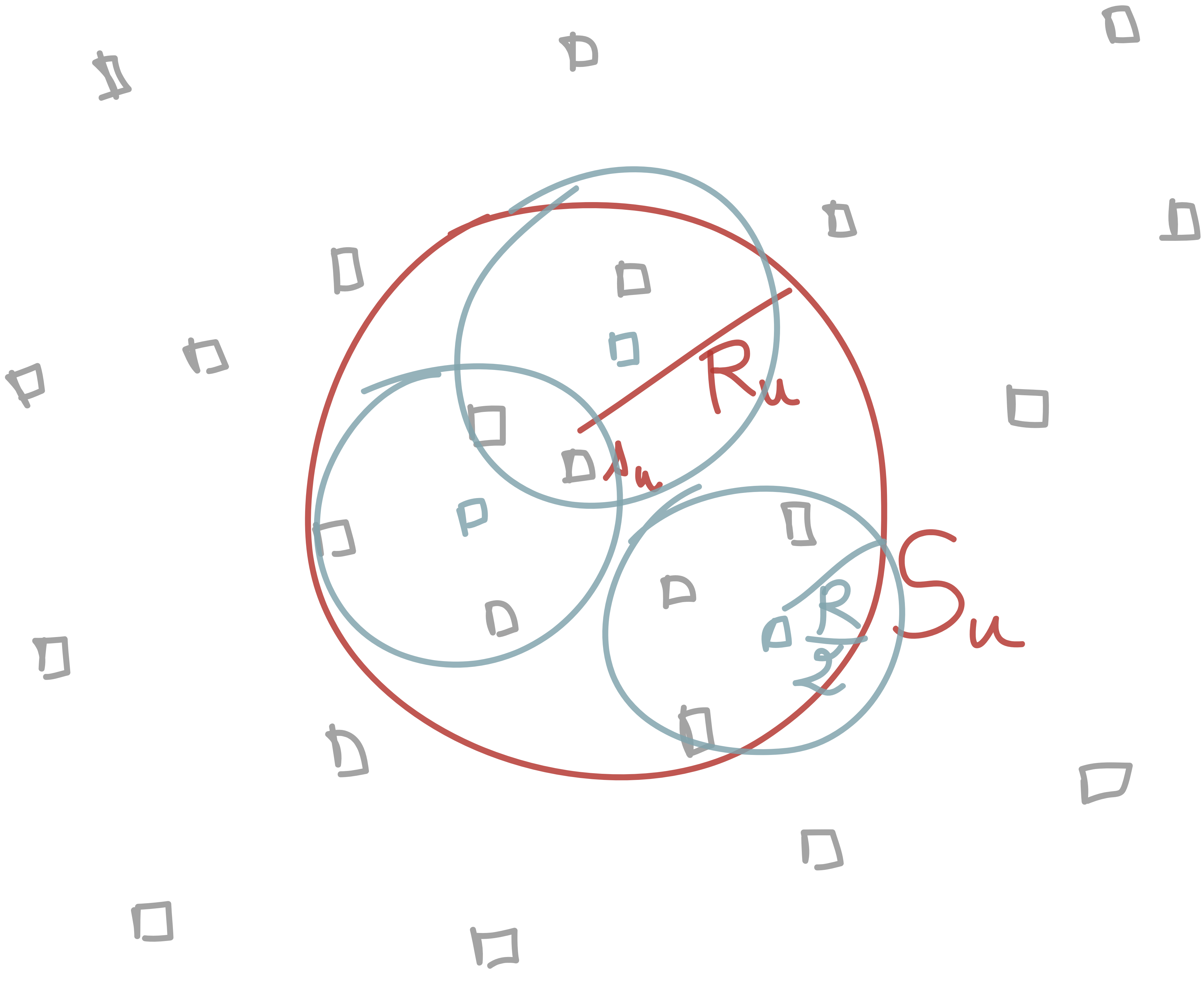
Table 1: User-specific values for topics.

Structured interests

Structured interests



Structured interests



Structured interests

- Subjects are in a metric space.
- B(s,R) is the ball of subjects at dist. ≤R from s.
- The interest set of each u is a ball B(s_u,R_u).

Sufficient conditions for optimality

- g-doubling : any B(s,R) is ⊆ in ≤g balls rad. R/2
- r-covering : ∀s∈S, ∃u dist(s,s_u)≤r and R_u≥r.
- (r,a)-sparsity : ∀s∈S, |B(s,r)|≤a
- r-interest-radius regularity : ∀u,v s.t. dist(s_u,s_v)<3R_u/2+r, we have R_v≥R_u/2+r

Sufficient conditions for optimality

- Prop : Under the previous assumptions, ∃G s.t. each u receives all s∈S_u and has indegree at most ga+g^2 log R_u/r.
- Optimal if ga+g^2 log R_u/r ≤ D_u for all u.

Sufficient conditions for stability

- Expertise-filtering rule : when u follows v, it receive only s s.t. dist(v,s)≤dist(u,v).
- Nearest subject first : when reconnecting, u gives priority to subjects closer to s_u, i.e. reconnected to get s∉R(u) iff no subject s' with dist(s_u,s') < dist(s_u,s) is lost.

Sufficient conditions for stability

- Th 2 : With expertise filtering and nearest-subject-first priority, if the metric satisfies the previous conditions on the metric, and $D_u \geq ga + g^2 \log R_u/r$ for all u, then selfish dynamics converge to a state where each user receives whole his interest set.
- Convergence is fast : logarithmic number of rounds.

Summary of results

Interests	Convergence	Price of Anarchy
Homogeneous	Yes (exp.)	Low (deg. ≥ 3)
Heterogeneous	No	High
Metric space	Yes (log.)	Opt. (log. deg.)

Conclusion / Perspectives

- Simple model with already complex dynamics.
- Structured interests with natural rules may explain tractability.
- TODO : study the structure of interests through real data.
- Better model spam: $cost(vu) = |S_v| / |S_v \cap S_u|$