PLANAR ROTATION SEQUENCES AND DOMAIN EXCHANGE

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(JOINT WORK WITH SHIGEKI AKIYAMA, ATTILA PETHŐ, AND WOLFGANG STEINER)

In [2] it was conjectured that all integer sequences $(a_k)_{k\in\mathbb{Z}}$ satisfying

$$0 \le a_{k-1} + \lambda a_k + a_{k+1} < 1 \quad (k \in \mathbb{Z})$$

for real λ with $|\lambda| < 2$ are periodic. This question arose in the study of shift radix systems.

The conjecture is trivially true for $\lambda = -1, 0, 1$. A computer assisted proof for $\lambda = \frac{1-\sqrt{5}}{2}$ was given by Lowenstein, Hatjispyros and Vivaldi [5], where also the solution for $\lambda = \frac{1+\sqrt{5}}{2}$ is mentioned. A short proof (without use of computers) of the latter case was given by the authors [1].

The proof in [5] is based on a torus map which is described in detail by Kouptsov, Lowenstein and Vivaldi [4] for all quadratic λ corresponding to rational rotations ($\lambda = \frac{\pm 1 \pm \sqrt{5}}{2}, \pm\sqrt{2}, \pm\sqrt{3}$), by heavy use of computers. Important related work is due to Adler, Kitchens and Tresser [3], Poggiaspalla [6], Vivaldi and Lowenstein [7] and others.

We present a survey on a new method similar to the one in [5]. It is based on the study of a piecewise affine torus map and allows proving the conjecture for quadratic λ corresponding to rational rotations and determining all possible period lengths.

References

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Date: October 27, 2006.