

HW3 Randomized Algorithms

MPRI 1.24 Tue. Dec. 2, 2014 - Due on Tue. Dec. 9, 2014



You are asked to complete the exercise marked with a [★] and to send me your solutions at:
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(or drop it in my mail box at the 4th floor of Sophie Germain) on Tue. Dec. 9, 2014.

■ **Exercise 1 (FPT algorithm for spotting k disjoint triangles).** [★]

Given $G = (V, E)$ an undirected graph ($n = |V|$ and $m = |E|$) and k an integer, we are looking for k vertex-disjoint triangles in G . Note that this problem is NP -complete when k is part of the input. We are looking for an algorithm of time complexity $O(f(k)n^a m^b)$ where the exponents a and b are constant, independent of k . Such an algorithm is called FPT for Fixed Parameter Tractable, which means that the complexity is a fixed-degree polynomial in the size of the input for any fixed value of the parameter k . Consider the following randomized algorithm:

Algorithm 1 FPT randomized algorithm for k disjoint triangles

- Choose independently for each vertex u a color $c_u \in \{1, \dots, 3k\}$ uniformly at random.
 - **return** "Yes" if there is a *colorful* solution, i.e. a set of k triangles whose $3k$ vertices use exactly once each color; **return** "I don't know" otherwise.
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► **Question 1.1)** Show that if G contains k disjoint triangles, then the probability that the algorithm answer "Yes" is at least e^{-3k} .

▷ Hint. use that: $k! \geq (k/e)^k$ for all k .

► **Question 1.2)** How many times should you run this algorithm to improve success probability to $1/2$?

In order to check whether a colorful solution exists, we propose to try all permutations π on $\{1, \dots, 3k\}$ and check if there is are triangles of colors $(\pi_1, \pi_2, \pi_3), \dots, (\pi_{3k-2}, \pi_{3k-1}, \pi_{3k})$.

► **Question 1.3)** Describe an algorithm that decides if such a collection of triangles exists. (Explicit the exact data structure you are using). What is the overall expected time complexity in k , n and m , of the algorithm that uses this method to return k disjoint triangles with probability at least $1/2$ if they exists in G ? What is the time complexity if k is fixed?